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PERCUBAAN STPM 2009

MATHEMATICS T(MATEMATIK T)

PAPER 1(KERTAS 1)

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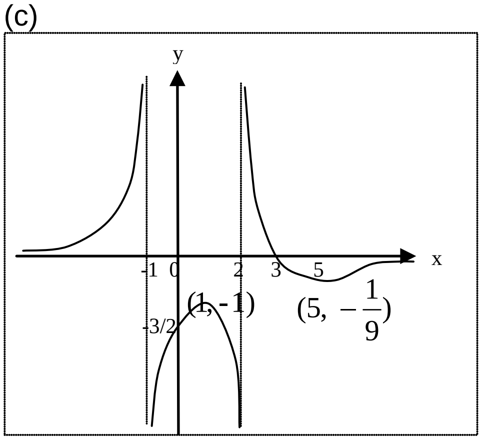
MARKING SCHEME

No.		Marks	
1 [4]	$\lvert x+y\,i + 3 - 4\,i \rvert = \lvert 3\,i \rvert$ $\sqrt{(x+3)^2 + (y-4)^2} = 3$ $x^2 + 6x + 9 + y^2 - 8y + 16 = 9$ $x^2 + y^2 + 6x - 8y + 16 = 0$	1 1 1 1	Using his in the modulus Equation correct. For squaring CA
2 [5]	$f(x) = x^3 - 2$ $f(x+\delta x) = (x+\delta x)^3 - 2$ $= x^3 + 3x^2\delta x + 3x(\delta x)^2 + (\delta x)^3 - 2$ $f\,'(x) = \lim\limits_{\delta x \to 0} \dfrac{(x+\delta x)^3 - x^3}{\delta x}$ $= \lim\limits_{\delta x \to 0} \dfrac{3x^2\delta x + 3x(\delta x)^2 + (\delta x)^3}{\delta x}$ $= 3x^2$ $f\,'(a) = 3a^2$	1 1 1 1 1	$f(x+\delta x) = (x+\delta x)^3 - 2$ correct formula substitute with limit Expand CA CA
3 [6]	For $\log_3(x^2 - 3x + 2)$ and $2\log_3(2x-1)$ to be defined, $x^2 - 3x + 2 > 0$ and $2x - 1 > 0$ $\Rightarrow \quad x < 1$ or $x > 2$, and $x > \dfrac{1}{2}$ $\Rightarrow \quad \dfrac{1}{2} < x < 1$ or $x > 2$ $\log_3(x^2 - 3x + 2) < \log_3 2(2x-1)^2$ $x^2 - 3x + 2 < 2(2x-1)^2$ $x < 0$ or $x > \dfrac{5}{7}$ $\therefore$ the solution set is $\left\{ x : \dfrac{5}{7} < x < 1 \text{ or } x > 2 \right\}$	1 1 1 1 1 1	both conditions (with "and") Correct way of using laws
4 [5]	. $-8 + 16 + 2a + b = 0$ $a^3 + 4a^2 - a^2 + b = a^3$ $2a - 3a^2 + 8 = 0$ $(3a + 4)(a - 2) = 0$ $a = -\frac{4}{3}$, $b = -\frac{16}{3}$ $a = 2$, $b = -12$	1 1 1 1 1	CA CA Solving

5 [6]	$S_{n-1} = \dfrac{a^{3-n}(b^{n-1} - a^{n-1})}{b-a}$	1	
	$T_n = S_n - S_{n-1} = \dfrac{a^{2-n}(b^n - a^n)}{b-a} - \dfrac{a^{3-n}(b^{n-1} - a^{n-1})}{b-a}$ $= \dfrac{a^{2-n}b^n - a^2 - a^{3-n}b^{n-1} + a^2}{b-a}$	1	Use his S_{n-1} In the formula
	$= \dfrac{a^{2-n}b^{n-1}(b-a)}{b-a}$	1	factorisation
	$= a^{2-n}\, b^{n-1}$.	1	
	$\dfrac{T_n}{T_{n-1}} = \dfrac{a^{2-n}b^{n-1}}{a^{3-n}b^{n-2}}$	1	use his to find r (do division)
	$= \dfrac{b}{a}$ (independent of n) $\Rightarrow$ the series is a geometric series.	1	with correct reason
6 [9]	$f(x) = x^3 - 10x^2 + ax + b$ $\because 3+\sqrt{2}$ is a zero, $\therefore$ $3-\sqrt{2}$ ia also a zero. $\left[x-(3+\sqrt{2})\right]\left[x-(3-\sqrt{2})\right] = x^2 - 6x + 7$	1	Conjugate must be correct
	$x^3 - 10x^2 + ax + b = (x^2 - 6x + 7)(x + \frac{b}{7})$	1	Use factorisation or long division
	Equating coefficient of x^2 : $\frac{b}{7} - 6 = -10 \Rightarrow b = -28$ Equating coefficient of x : $7 - \frac{6b}{7} = a \Rightarrow a = 31$	1	Method to find a or b
	$a = 31,\ b = -28$	1	both a and b correct
	(a) $f(x) = 12 - 3x$		
	$x^3 - 10x^2 + 31x - 28 = 12 - 3x$ $(x-4)(x^2 - 6x + 10) = 0$ $x = 4$ or $x^2 - 6x + 10 = 0$	1	factorisation
	For $x^2 - 6x + 10 = 0$ *	1	* followed by **
	Discriminant $= (-6)^2 - 4(1)(10) = -4 < 0$ ** Hence, the only real root is 4.	1	Conclusion

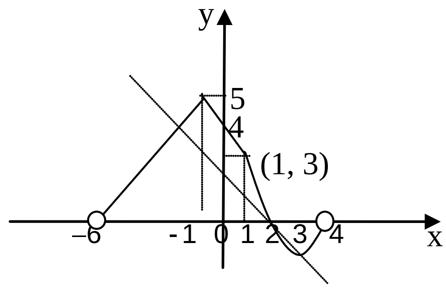
	(b) $x^3-10x^2+31x-28=(x^2-4)Q(x)+Ax+B$ When $x=2$, $2=2A+B$........(1) When $x=-2$, $-138=-2A+B$.....(2) $(1)+(2)$, $2B=-136 \Rightarrow \text{B}=-68$ $(1)-(2)$, $4A=140 \Rightarrow A=35$ $\therefore$ the remainder is $35x-68$.	1 1	Method CA
7 [9]	7(a) $\sqrt{1-3x}=(1-3x)^{\frac{1}{2}}$ $=1+\frac{1}{2}(-3x)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(-3x)^2+\frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})}{3!}(-3x)^2+\ldots$ $=1-\frac{3}{2}x+\frac{9}{8}x^2-\frac{9}{16}x^3+\ldots$ Range : $\left\{x:-\frac{1}{3}<x<\frac{1}{3}\right\}$ (b) $\left(x-\frac{1}{3x^2}\right)^{24}$ $U_{r+1}=\binom{24}{r}x^{24-r}\left(-\frac{1}{3x^2}\right)^r$ $=\binom{24}{r}(-1)^r\left(\frac{1}{3}\right)^r x^{24-3r}$ $24-3r=0$ $r=8$ The term independent of $x=\binom{24}{8}(-1)^8\left(\frac{1}{3}\right)^8x^0$ $=\frac{81719}{729}$	1 1, 1 1 1 1 1 1 1	 2 correct (1) 3 correct(1+1) The term in x must be correct His = 0
8 [10]	$y=\frac{x-3}{(2-x)(x+1)}=\frac{x-3}{-x^2+x+2}$ (a) Asymptotes : $x=2$, $x=-1$, $\text{y}=0$. $\frac{dy}{dx}=\frac{(-x^2+x+2)-(x-3)(-2x+1)}{(-x^2+x+2)^2}$ $=\frac{x^2-6x+5}{(-x^2+x+2)^2}$	 2 1	 2 correct = 1 3 correc = 2 formula

	$\frac{dy}{dx}=0$ $\frac{x^2-6x+5}{\left(-x^2+x+2\right)^2}=0$	1	His = 0
	$x^2-6x+5=0$ $(x-1)(x-5)=0$ $x=1,\ 5.$		
	(b) Turning points are $(1,\ -1)$ and $(5,-\frac{1}{9})$.	1	Correct answer in coordinate form
	$\frac{d^2y}{dx^2}=\frac{(2x-6)(-x^2+x+2)^2-(x^2-6x+5)(2)(-x^2+x-2)(-2x+1)}{(-x^2+x+2)^4}$ $=\frac{(2x-6)(-x^2+x+2)-(x^2-6x+5)(2)(-2x+1)}{(-x^2+x+2)^3}$	1	method
	Point (1, -1), $\frac{d^2y}{dx^2}<0 \Rightarrow$ local maximum point Point $\left(5,\ -\frac{1}{9}\right)$, $\frac{d^2y}{dx^2}>0 \Rightarrow$ local minimum point	1	nature of both the turning points correct
	(c)  	1 1 1	all parts correct points asymptotes and perfect
9 [10]	x(8m – m^2 x) = 16 m^2 x^2 – 8mx + 16 = 0	1	quadratic eq.
	b^2 – 4ac = 64m^2 – 4 (m^2)(16) = 0	1	b^2-4ac=0 or other method
	$\therefore$ m^2 x + y – 8m = 0 is a tangent to xy = 16.	1	Conclusion
	Eqn. of OR is y = $\frac{1}{m^2}$ x …………(1) m^2 x + y – 8m = 0 ………….(2)	1	Equation of OR
	(1) $\rightarrow$ (2) m^2 x + $\frac{1}{m^2}$ x – 8m = 0 x = $\frac{8m^3}{1+m^4}$	1	substitution

	$y = \dfrac{8m}{1+m^4}$	1	x or y correct
	R ($\dfrac{8m^3}{1+m^4}$, $\dfrac{8m}{1+m^4}$)	1	Coordinates R
	$m^2 = \dfrac{x}{y}$	1	m or m^2 in terms of x and y.
	$(m^2 x + y)^2 = 64m^2$	1	elimination of m
	$(\dfrac{x}{y} \times x + y)^2 = 64\,(\dfrac{x}{y})$		
	$(x^2 + y^2)^2 = 64xy$	1	
10 [11]	$\lvert \mathbf{A} \rvert = k\begin{vmatrix} -3 & 1 \\ 1 & 2 \end{vmatrix} - 1\begin{vmatrix} 2 & 1 \\ 2 & 2 \end{vmatrix} - 1\begin{vmatrix} 2 & -3 \\ 2 & 1 \end{vmatrix}$	1	CA
	$= k(-6-1)-(4-2)-(2+6)$ $= -7k - 2 - 8$ $= -7k - 10$		
	$-7k - 10 = -17$ $7k = 7$ $k = 1$	1	
	$\mathbf{BC} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 5 & 6 \end{pmatrix}\begin{pmatrix} 0 & -17 & 0 \\ -66 & -38 & 3 \\ 51 & 34 & 0 \end{pmatrix}$	*1	( )( ) is seen
	$= \begin{pmatrix} 21 & 9 & 6 \\ 6 & -12 & 9 \\ -24 & -3 & 15 \end{pmatrix}$	1	CA (if 1 is not obtained here, multiplication process must be seen to obtain *1)
	$\mathbf{ABC} = \begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 1 \\ 2 & 1 & 2 \end{pmatrix}\begin{pmatrix} 21 & 9 & 6 \\ 6 & -12 & 9 \\ -24 & -3 & 15 \end{pmatrix}$	1	A(*) * his BC
	$= \begin{pmatrix} 51 & 0 & 0 \\ 0 & 51 & 0 \\ 0 & 0 & 51 \end{pmatrix}$		
	$\mathbf{ABC} = 51\,\mathbf{I}$	1	
	$\mathbf{A}^{-1} = \frac{1}{51}\begin{pmatrix} 21 & 9 & 6 \\ 6 & -12 & 9 \\ -24 & -3 & 15 \end{pmatrix}$	1	His BC
	$= \begin{pmatrix} 7/17 & 3/17 & 2/17 \\ 2/17 & -4/17 & 3/17 \\ -8/17 & -1/17 & 5/17 \end{pmatrix}$	1	CA
	$\begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 1 \\ 2 & 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ -3 \\ 25 \end{pmatrix}$	1	CA
	$\mathbf{A}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ -3 \\ 25 \end{pmatrix}$		

	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{51}\begin{pmatrix} 21 & 9 & 6 \\ 6 & -12 & 9 \\ -24 & -3 & 15 \end{pmatrix}\begin{pmatrix} -1 \\ -3 \\ 25 \end{pmatrix}$	1	His inverse
	$= \frac{1}{51}\begin{pmatrix} 102 \\ 255 \\ 408 \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \\ 8 \end{pmatrix}$		
	x = 2. y = 5, z = 8	1	CA
11 [11]	a) $\frac{5x^2+4x+12}{(x+2)(x^2+4)} \equiv \frac{A}{x+2} + \frac{Bx+C}{x^2+4}$	1	CA
	$5x^2 + 4x + 12 = A(x^2+4) + (Bx+C)(x+2)$ $x=-2$, $20 - 8+12 = 8A$ $A = 3$ coef. of x^2, $5 = A + B$ $B = 2$ $x = 0$, $12 = 4A + 2C$ $C = 0$	1	Method to find A, B & C
	$\therefore \frac{5x^2+4x+12}{(x+2)(x^2+4)} \equiv \frac{3}{x+2} + \frac{2x}{x^2+4}$	1	CA
	$\int_1^3 \frac{5x^2+4x+12}{(x+2)(x^2+4)} dx$ $= \int_1^3 \frac{3}{x+2} + \frac{2x}{x^2+4} dx$		
	$= [3\ln(x+2) + \ln(x^2+4)]_1^3$	1, 1	First 1, ln is seen 2nd 1, all correct
	$= 3\ln(5) + \ln(13) - 3\ln(3) - \ln(5)$	1	his substitution
	$= \ln\frac{325}{27}$	1	CA
	b) $h = \frac{2-1}{4} = 0.25$	1	CA
	(table below)	1	All correct
	$\int_1^2 \ln(1+x^2)\ dx = \frac{0.25}{2}[0.6931+1.6094 + 2(0.9410+1.1787+1.4018)]$	1	Correct formula with his value
	$= 1.168$	1	CA

x	y
1	0.6931
1.25	0.9410
1.5	1.1787
1.75	1.4018
2	1.6094

12 [14]	(a) $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}(5-\lvert x-1\rvert) = 3$	1	CA
	$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}(x^2-6x+8) = 3$	1	CA
	$f(1) = 1-6+8 = 3$ $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1).$ * * $\Rightarrow f$ is continuous at $x = 1$.	1	With reason **
	(b) 	1, 1	Parts of $y = 5-\lvert x+1\rvert$
		1	parabola
		1	points, $\circ$ is marked at end-points
	(c) $y = 9-18+8 = -1$	1	method to find minimum point
	Range of $\{y: -1 \le y \le 5\}$	1	CA
	(d) $5-(-x-1) = 2-x$	1	CA
	$x-2 = x^2-6x+8$	1	CA
	$x = -2,\ 2,\ 3,$	1	All correct
	$\{x: -2 < x < 2\} \cup \{x: 3 < x < 4\}$ If other method is used, mark accordingly.	1,1	Correct line is see(if no line is seen, no mark) Ignore shading If 1 set is correct, I mark only.