Subject : Date :

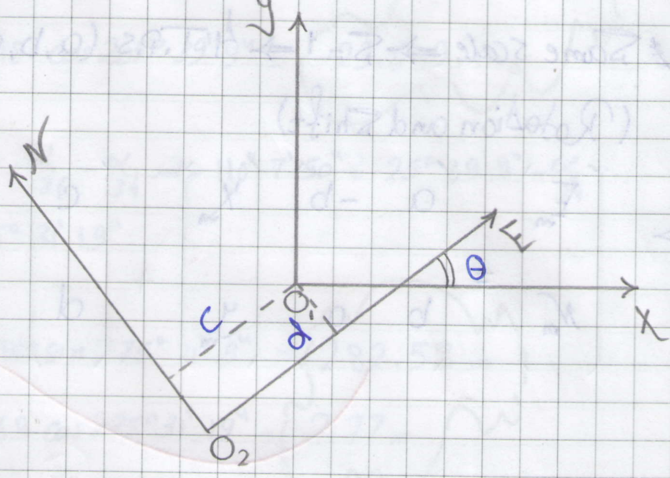
Assignment 3 :-

[1] Case a :-

* Trans. par. ⇒ T.p.

⇒ There are 4 T.p.s (Rotation, shift, Scale)

(a , b) , (c , d) and



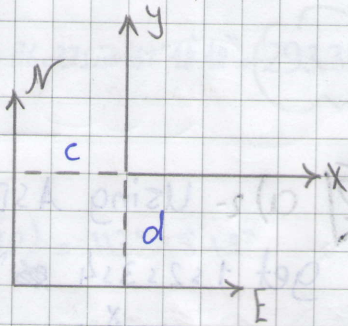
* $a = S\cos\theta$, $b = S\sin\theta$, for point M in two different 2D co-od sys. (E,N) and (X,y) ⇒ $E_M = S X_m \cos\theta + S y_m \sin\theta$, $N_m = -S X_m \sin\theta + S y_m \cos\theta$

$$\Rightarrow \begin{bmatrix} E_m \\ N_m \end{bmatrix} = \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} X_m \\ y_m \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix}$$

assuming θ is anticlockwise.

Case b :- parallel ⇒ $\theta = 0$

⇒ 3 T.p.s (a, c, d) ⇒ only shift and Scale

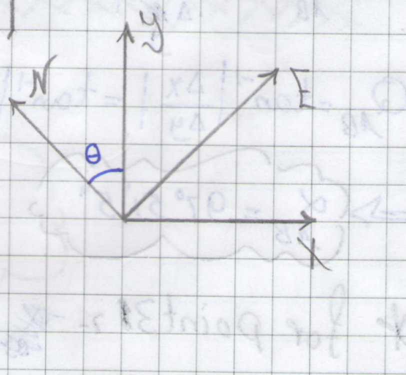


$$\begin{bmatrix} E_m \\ N_m \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} \begin{bmatrix} X_m \\ y_m \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix}$$

---

case C :- Same origin ⇒ $c = d = $ zero

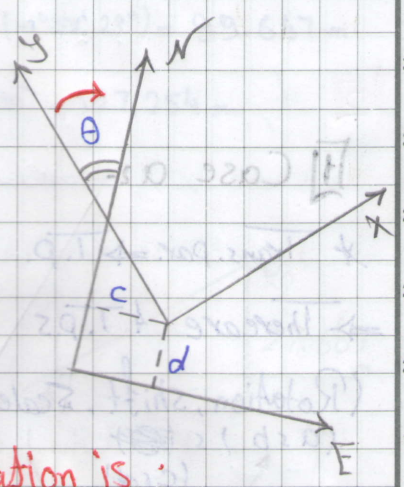
2 T.p.s ⇒ a , b (Rotation and Scale)



$$\begin{bmatrix} E_m \\ N_m \end{bmatrix} = \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} X_m \\ y_m \end{bmatrix}$$

Case d :- for a 2-D co-ordinate system :-

* Same scale → $S = 1$ → 4 T.P.S (a, b, c, d)
(Rotation and Shift)

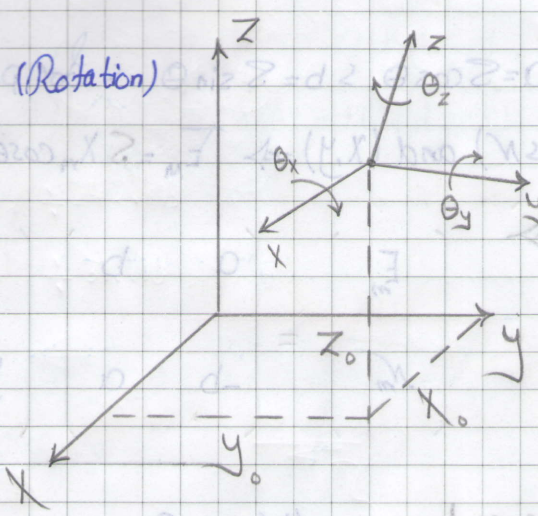


$$\Rightarrow \begin{pmatrix} E_m \\ N_m \end{pmatrix} = \begin{pmatrix} a & -b \\ b & a \end{pmatrix} \begin{pmatrix} X_m \\ y_m \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix}$$

Rotation is clockwise.

3-D :- 6 unknowns :- $\theta_x, \theta_y, \theta_z$ (Rotation)
, X_0, y_0, Z_0 for Shift.

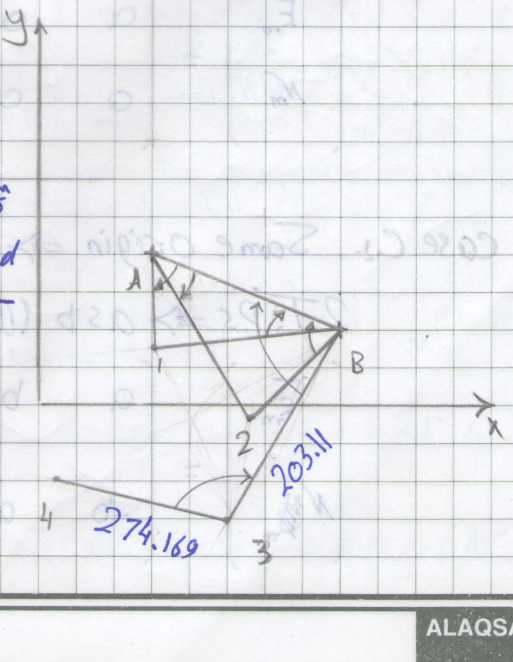
$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = SR \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix}$$



[2] a) :- Using A, B xy co-ods, H_z angles, H_z dists.
get 1, 2, 3, 4 xy co-ods.

→ $\cancel{Q_{AB} = \tan^{-1}\left|\frac{\Delta X}{\Delta Y}\right| = \tan^{-1}\left| \right|}$

$Q_{AB} = \tan^{-1}\left|\frac{\Delta x}{\Delta y}\right| = \tan^{-1}\left|\frac{386.33}{-53.274}\right| = 82° 8' 55"$
2nd quad



→ $\alpha_{AB} = 97° 51' 5"$

* for point 3 :- $\cancel{\alpha_{B3}}$ $3\hat{B}A = \alpha_{BA} - \alpha_{B3}$

Subject : Date :

$\Rightarrow 72°11'56" = 277°51'5" - \alpha_{B3} \Rightarrow \alpha_{B3} = 205°39'9"$

$\Rightarrow X_3 = X_B + l_{B3} \sin\alpha_{B3} \Rightarrow X_3 = 643.408 + 203.11 \sin 205°39'9" = 555.479\ m$

$Y_3 = Y_B + l_{B3} \cos\alpha_{B3} \Rightarrow Y_3 = 159.481 \cos 205°39'9" = -23.609\ m$

* for point 4 :- $\alpha_{34} = ? \Rightarrow 4\hat{3}B = \alpha_{3B} - \alpha_{34} \Rightarrow 110°7'50" = 25°39'9" - \alpha_{34}$

$\Rightarrow \alpha_{34} = -84°28'41" + 360° = 275°31'19"$

$\Rightarrow X_4 = X_3 + l \sin\alpha = 555.479 + 274.169 \sin 275°31'19" = 282.58\ m$

$, Y_4 = Y_3 + l\cos\alpha = -23.609 + 274.169 \cos 275°31'19" = 2.77\ m$

* for point 1 :- $\Delta AB1$:- $l_{AB} = \sqrt{\Delta X^2 + \Delta Y^2} = 389.985\ m$

$\Rightarrow A\hat{1}B = 180 - (1\hat{B}A + B\hat{A}1) = 98°53'56"$

Sin rule :- $\dfrac{389.985}{\sin 98°53'56"} = \dfrac{A1}{\sin 17°8'21"} \Rightarrow A1 = 116.33\ m$

$B\hat{A}1 = \alpha_{A1} - \alpha_{AB} \Rightarrow \alpha_{A1} = 161°48'48"$

$\Rightarrow$ line A1 $\Rightarrow X_1 = X_A + l \sin\alpha_{A1} \Rightarrow X_1 = \cancel{257}\ 257.079 + 116.33 \sin 161°48'48" = 293.387\ m$

$, Y_1 = 212.755 + 116.33 \cos 161°48'48" = 102.236\ m$

* for point 2 :- $\Delta A2B$:- $A\hat{2}B = 180 - (2\hat{B}A + B\hat{A}2) = 113°18'18"$

Sin rule :- $\dfrac{389.985}{\sin 113°18'18"} = \dfrac{B2}{\sin 19°9'18"} \Rightarrow B2 = 139.3\ m$

$2\hat{B}A = \alpha_{BA} - \alpha_{B2} \Rightarrow \alpha_{B2} = 230°18'41"$

$\Rightarrow X_2 = 643.408 + 139.3 \sin 230°18'41" = 536.2\ m$

$, Y_2 = 159.481 + 139.3 \cos 230°18'41" = 70.52\ m$

ALAQSA

b) 2- 2 common points are needed $\Rightarrow$ points 1 , 2 .

from the xy to the EN sys.

$E = x S \cos\theta + y S \sin\theta \Rightarrow E = ax + by + c$

θ is anticlockwise

$N = -x S \sin\theta + y S \cos\theta \Rightarrow N = -bx + ay + d$

	1	2
x	293.387	536.2
y	102.236	70.52
E	930.656	1161.735
N	458.633	539.721

* E_1: $930.656 = a(293.387) + b(102.236) + c$ ①

* N_1: $458.633 = -b(293.387) + a(102.236) + d$ ②

* $1161.735 = a(536.2) + b(70.52) + c$ ③

* $539.721 = -b(536.2) + a(70.52) + d$ ④

$\int 3 - \int 1 \Rightarrow 231.079 = 242.813a - 31.716b \quad \div 242.813 \Rightarrow .95 = a - .13b$

$\int 4 - \int 2 \Rightarrow 81.088 = -242.813b - 31.716a \quad \div 31.716 \Rightarrow 2.556 = -7.655b - a$

$3.506 = -7.785b$

$b = -.45$, $a = .89$

$\therefore S = .997$, $\theta = -26° 49' 19'' = 26° 49' 19''$ clockwise.

$\Rightarrow c = 715.55\ m$, $d = 235.62\ m$

~~$a = .89 = a$~~

~~$b = .45 = -b$~~

c) 2- for 3 2- $E_3 = ax_3 - by_3 + c$

$= .89(555.479) - .45(-23.609) + 715.55 = 1220.55\ m$

, $N_3 = .45(555.479) + .89(-23.609) + 235.62 = 464.57\ m$

عوضه في القانون عادي جداً

for 4 2-

$E_4 = .89(282.58) - .45(2.77) + 715.55 = 965.799\ m$

$N_4 = .45(282.58) + .89(2.77) + 235.62 = 365.246\ m$

$\boxed{3}$:- $Q_{AB} = \tan^{-1}\left|\frac{\Delta E}{\Delta N}\right| = \tan^{-1}\left|\frac{\Delta x}{\Delta y}\right| = \tan^{-1}\left|\frac{60.09}{-42.124}\right| = 54°58'8''$ 2nd quad

EN زي XY اعتبر
بالضبط .

$\Rightarrow \alpha_{AB} = 125°1'52''$, $HcR\,B = 100°$

$\therefore$ Rotate H_z circle $25°1'52''$ to set orientation.

* add $25°1'52''$ to each HcR $\Rightarrow$ get Bearings.

Check:- $2\hat{A}B = HcR_{AB} - HcR_{A2} = 55°20'19'' = \alpha_{AB} - \alpha_{A2} \Rightarrow \alpha_{A2} = 69°41'6''$

, $HcR_{A2} = 44°39'21'' \Rightarrow$ ~~α is~~ difference $= 25°1'52''$

* Bearings:- $A1 = 51°7'32''$, $A2 = 69°41'6''$, $A3 = 42°5'4''$
$A4 = 30°55'46''$

* Vertical angles:- $2 = 90° - 88° = 2°$, $1 = 0°30'$, $3 = 0°$, $4 = 1°$

* $\overset{x}{E_n} = \overset{x}{E_o} + S\cos V \sin HcR$, $\overset{y}{N_n} = \overset{y}{N_o} + S\cos V \cos HcR$

Bearing

a):-

$X_1 = 416.875 + 77.894\cos 30' \cdot \sin 51°7'32'' = 477.5\,m$

$y_1 = 257.094 + 77.894\cos 30' \cdot \cos 51°7'32'' = 305.979\,m$

$X_2 = 416.875 + 105.372\cos 2° \cdot \sin 69°41'6'' = 515.63\,m$

$y_2 = 257.094 + 105.372\cos 2° \cdot \cos 69°41'6'' = 293.655\,m$

$X_3 = 416.875 + 201.508\cos 0° \cdot \sin 42°5'4'' = 551.93\,m$

$y_3 = 257.094 + 201.508\cos 0° \cdot \cos 42°5'4'' = 406.64\,m$

$X_4 = 416.875 + 190.042\cos 1° \cdot \sin 30°55'46' = 514.538\,m$

$y_4 = 257.094 + 190.042\cos 1° \cdot \cos 30°55'46' = 420.087\,m$

$$\alpha_{CD} = \theta_{CD} = \tan^{-1}\left|\frac{52.857}{45.634}\right| = 49°\,11'\,40'' \Rightarrow$$ Then Rotate H_z circle $49°\,11'\,40''$ to set orientation.

* Bearings :- C3 = 133° 41' 6" , C4 = 103° 49' 45" , CX = 286° 49' 10"

* Vertical angles :- C3 = −10' , C4 = 10' , CX = 6'

$$E_n = E_o + S\cos V \sin HCR \qquad N_n = N_o + S\cos V . \cos HCR$$

b) :- $E_3 = 2777.317 + 105.372\cos(-10')\sin 133°\,41'\,6'' = \boxed{2853.516\,m}$

$N_3 = 2172.561 + 105.372\cos(-10')\cos 133°\,41'\,6'' = \boxed{2099.78\,m}$

$E_4 = 2777.317 + 77.894\cos 10' \sin 103°\,49'\,45'' = \boxed{2852.95\,m}$

$N_4 = 2172.561 + \cancel{105.372}\ 77.894\cos 10' \cos 103°\,49'\,45'' = \boxed{2153.94\,m}$

$E_x = 2777.317 + 42.8\cos 6' \sin 286°\,49'\,10'' = \boxed{2736.348\,m}$

$N_x = 2172.561 + 42.8\cos 6' \cos 286°\,49'\,10'' = \boxed{2184.945\,m}$

c) :- 3 , 4 are 2 common points.

$$E = aX + bY + C \qquad , \qquad N = -bX + aY + d$$

E_3 ⇒ $2853.516 = a(551.93) + b(406.64) + C$ (1) $2852.95 = a(514.538) + b(420.087) + C$ (3)

N_3 $\cancel{2172.561}$ 2099.78 $= -b(551.93) + a(406.64) + d$ (2) $2153.94 = -b(514.538) + a(420.087) + d$ (4)

(1) − (3) ⇒ $.566 = 37.392a - 13.447b \ \div 13.447 \Rightarrow .042 = 2.78a - b$

(4) − (2) ⇒ $54.16 = 37.392b + 13.447a \ \div 37.392 \Rightarrow 1.448 = b + .3596a$

$1.49 = 3.139a$ ⇐

$\boxed{a = .47}$, $\boxed{b = .618}$

$\theta = 52°\,44'\,47''$, $\boxed{C = 2351.5\,m}$, $\boxed{d = 2274.48\,m}$

$S = .776$

d) :- $E_1 = .47(477.5) + .618(305.979) + 2351.5 = \boxed{2765.02\,m}$

$N_1 = -.618(477.5) + .47(305.979) + 2274.48 = \boxed{2123.195\,m}$

Subject : Date :

$\Delta E_{x1} = 28.672\ m$, $\Delta N_{x1} = -61.75$

$l_{x1} = 68.08\ m$

---

4 Check using lengths:- $\underline{AB}$:- $\Delta E = 249.52$, $\Delta N = 1173.899 \Rightarrow l = 1200.1246$

$\Delta x = 41.884$, $\Delta y = 1199.406 \Rightarrow l = 1200.1370$

$\underline{BC}$:- $\Delta E = 981.782$, $\Delta N = 1413.725 \Rightarrow L = \boxed{1721.1955}$ Point C

$\Delta x = 657.671$, $\Delta y = 1348.426 \Rightarrow L = \boxed{1500.26125}$

$\underline{AD}$:- $\Delta E = 1733.006$, $\Delta N = 1097.361 \Rightarrow L = 2051.22182$

$\Delta x = 1516.14$, $\Delta y' = 1381.628 \Rightarrow L = 2051.23778$

Check again for C:- $\underline{CD}$:- $\Delta E = 501.664$, $\Delta N = -1490.273 \Rightarrow \boxed{L = 1572.4}$

$\Delta x = 816.585$, $\Delta y = -1166.204 \Rightarrow L = \boxed{1423.672}$

Obviously it's point C

⇒ Get the Tranformation parameters avoiding using station C.

* Using A&B :- $E = ax + by + c$ $N = -bx + ay + d$

$\Rightarrow 650002.702 = a(0) + b(0) + c \Rightarrow c = 650002.702$

Similarly $d = 810004.443$

$650252.222 = a(41.884) + b(1199.406) + 650002.702$

$\Rightarrow 249.52 = 41.884a + 1199.406b \div 1199.406 \Rightarrow .208 = .034a + b$

$811178.342 = -b(41.884) + a(1199.406) + 810004.443$

$\Rightarrow 1173.899 = 1199.406a - 41.884b \div 41.884 \Rightarrow 28.027 = 28.63a - b$

$a = 1.015$, $b = 1.038$ $\Leftarrow 28.235 = 28.664a$

Subject : Date :

* لا تشتغل على الـ false Northing ، false Easting عادي.

* لو نظام إحداثيات local (افتراضي) فعلاقة الأطوال بينه وبين

نظام آخر متغيرتش في حاجة. (الأطوال متساوية).

$\boxed{5}$ case 1:- let $x_1, y_1 \Rightarrow x, y$ and $x_2, y_2 \Rightarrow E, N$

* $E = ax + by + c$, $N = -bx + ay + d$

$6000 = a(5000) + b(10000) + c$ (1) $6707.107 = a(5707.107) + b(10707.107) + c$ (3)

$11000 = -b(5000) + a(10000) + d$ (2) $11707.107 = -b(5707.107) + a(10707.107) + d$ (4)

(3) - (1) $\Rightarrow 707.107 = 707.107a + 707.107b \div 707.107 \Rightarrow 1 = a + b$

$\Rightarrow 2 = 2a$

(4) - (2) $\Rightarrow 707.107 = -707.107b + 707.107a \div 707.107 \Rightarrow 1 = -b + a$

$d = \boxed{1000} \Leftarrow c = \boxed{1000} \Leftarrow \boxed{b = 0} \Leftarrow \boxed{a = 1}$

case 2:- $5000 = a(5000) + b(10000) + c$ (1) $5965.926 = a(5707.107) + b(10707.107) + c$ (3)

$10000 = -b(5000) + a(10000) + d$ (2) $10258.819 = -b(5707.107) + a(10707.107) + d$ (4)

(3) - (1) $\Rightarrow 965.926 = 707.107a + 707.107b \div 707.107$, $a = .866$, $b = .5$

(4) - (2) $\Rightarrow 258.819 = -707.107b + 707.107a \div 707.107$, $c = -4329.98$, $d = 3840.01$