CE6501- STRUCTURAL ANALYSIS –CLASSICAL METHODS

(FOR V – SEMESTER)

UNIT – I(DEFLECTIONS OF DETERMINATE STRUCTURES)

UNIT – I

TWO MARKS QUESTIONS AND ANSWERS

Necessary to compute deflections in structures

Computation of deflection of structures is necessary for the following reasons:

(i) If the deflection of a structure is more than the permissible, the structure will not look aesthetic and will cause psychological upsetting of thje occupants.

(ii) Excessive deflection may cause cracking in the materials attached to the structure. For example, if the deflection of a floor beam is excessive, the floor finishes and partition walls supported on the beam may get cracked and unserviceable.

Cambering technique, in structures

Cambering is a technique applied on site, in which a slight upward curve is made in the structure / beams during construction, so that it will straighten out and attain the straight shape during loading. This will considerably reduce the downward deflection that may occur at later stages.

Four methods used for the computation of deflections in structures.

(i) Virtual work method – Dummy unit load method
(ii) Strain energy method
(iii) Willot Mohr diagram method
(iv) Method of elastic weights

Difference between strain energy method and unit load method in the determination of deflection of structures.

In strain energy method, an imaginary load P is applied at the point where the deflection is desired to be determined. P is equated to Zero in the final step and the deflection is obtained.

In unit load method, an unit load (instead of P) is applied at the point where the deflection is desired.

Assumptions made in the unit load method

Assumptions made in unit load method are

1. The external and internal forces are in equilibrium
2. Supports are rigid and no movement is possible.
3. The material is strained well within elastic limit.

Equation that is used for the determination of deflection at a given point i in beams and frames.

Deflection at a point i is given by,

$$\delta_i = \int_0^l \frac{M_x m_x \, dx}{EI}$$

Where M_x = moment at a section X due to the applied loads

M_x = moment at a section X due to unit load applied at the point i and in the direction of the desired dicplacement
EI = flexural rigidity

Principle of Virtual work.

It states that the work done on a structure by external loads is equal to the internal energy stored in a structure ($U_e = U_i$)

Work of external loads = work of internal loads

Strain energy stored in a rod of length l and axial rigidity AE to an axial force P

Strain energy stored

$$U = \frac{P^2 L}{2AE}$$

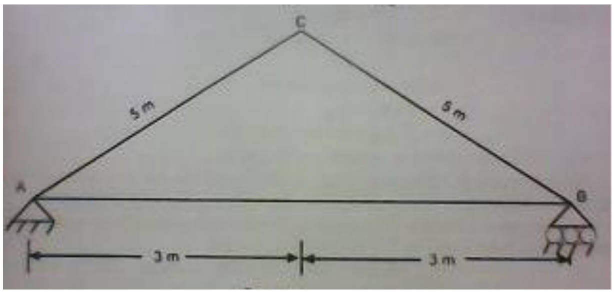
Virtual work.

The term virtual work means the work done by a real force acting through a virtual displacement or a virtual force acting through a real displacement. The virtual work is not a real quantity but an imaginary one.

Procedure involved in the deflection of pin jointed plane frames.

1. Virtual forces k: Remove all the real loads from the truss. Place a unit load on the truss at the joint and in the direction of the desired displacement. Use the method of joints or the method of sections and calculate the internal forces k in each member of the truss.
2. Real forces F: These forces arre caused only by the real loads acting on the truss. Use the method of sections or the method of joints to determine the forces F in each member.
3. Virtual work equation: Apply the equation of virtual work, to determine the desired displacement.

In the truss shown in fig. no load acts. The member AB gets 4mm too short. The cross sectional area of each member is A = 300 mm^2 and E = 200 GPa. Determine the vertical displacement of joint C.



Solution:

Virtual forces, k:

Since the vertical displacement of joint C is required, a vertical force of 1 KN is applied at C. The force k in each member is determined as below:

By symmetry, $R_A = R_B = ½$

Joint A: $\sum V = 0$ gives

$K_{AC} \cos 36^\circ\, 52' + ½ = 0$

$K_{AC} = 0.625$ kN (comp)

$\sum H = 0$ gives

$k_{AB} + k_{AC} \cos 53^\circ\, 08' = 0$

$k_{AB} = 0.375$ kN (tensile)

Joint B: $\sum V = 0$ gives

$K_{BC} \cos 36^\circ\, 52' + ½ = 0$

$K_{BC} = 0.625$ kN (comp)

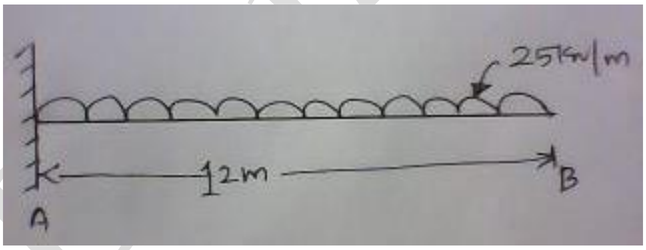
Member AB undergoes a deformation, $\Delta L = 0.004$ m

$\Delta = \sum(k.\Delta L)$

$(\Delta_C)_V = (0.375)\,(-0.004) + 0 + 0 = -0.0015\text{m} = -1.5$ mm

The negative sign indicates that joint C displaced upward, i.e. opposite to the 1 kN vertical load.

Using the method of virtual work, determine the vertical displacement of point B of the beam shown in fig. Take E = 2x 10^5 MPa and I = 825x 10^7 mm^4.



Solution:

$$\Delta = \int_0^l \frac{mMdx}{EI}$$

Virtual moment, m. Remove the external load. Apply a unit vertical load at B. Consider a section XX at a distance x from B.

$m = -1 * x$ (Hogging moment)

Real moment, M. Using the same x co-ordinate, the internal moment (due to the given loading) M is formulated as,

$M = -25. *.x^2/2$

$M = -12.5x^2$

Virtual work equation.

$$(\Delta_B)_V = \int_0^l \frac{mMdx}{EI}$$

$$= \int_0^{12} \frac{(-1 * x)(-12.5x^2)dx}{EI} = \int_0^{12} \frac{12.5x^3}{EI} dx$$

$$= \left[\frac{12.5x^4}{EIx4} \right]_0^{12} = \frac{12.5x12^4}{EIx4} = \frac{64800}{EI}$$

$$= \frac{64800}{2 * 10^8 \ x825x10^{-5}} = 0.0393m \ (or) \ 39.3 \ mm$$

Hence the vertical displacement of point B = 39.3 mm

Table shows the lengths and deformations of the members of the cantilever truss, shown in fig. Construct a Williot' diagram and tabulate the displacement of nodes.

Member	Length (mm)	Elongation (mm)
AC	6225	15.0
AD	4242	4.0
BD	4242	-10.5
DC	4242	-12.0

Solution:

Fig. shows the Willot's diagram of displacements. Table shows the displacements

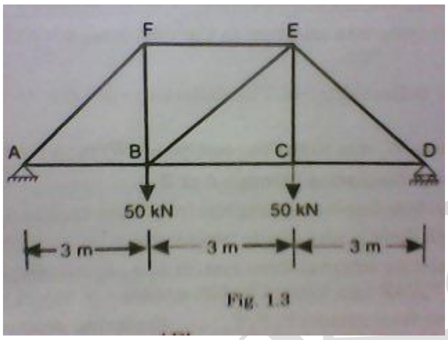
Node	Displacements (mm)	
	X	Y
C	-15.0	-47.6
D	-5.2	-11.5

1. Determine the vertical displacement of joint C of the steel truss shown in fig. The cross sectional area of each member is A = 400 mm² and E = 2*10⁵ N/mm².

Solution:

$$\Delta = \sum \frac{kFL}{AE}$$

Virtual forces k. Remove all the (external) loads and apply a unit vertical force at joint C of the truss. Analyze the truss using the method of joints.



Take moments about D, V_A x 9-1 x3 =0

V_A x 9 = 1*3, V_A = 1/3 kN

V_D = Total load – V_A = 1-1/3 = 2/3 kN

Joint A: Initially assume all forces to be tensile.

$$\sum V = 0 \ \ gives$$

$$k_{AF} \cos 45° + 1/3 = 0$$

$$k_{AF} \cos 45° = -1/3$$

$$k_{AF} = -\frac{1}{3\cos 45°} = -0.47\ kN$$

$$k_{AF} = 0.471\ kN(comp)$$

$$\sum H = 0\ gives$$

$$k_{AF} \cos 45° + k_{AB} = 0$$

$$(-0.471)\cos 45° + k_{AB} = 0$$

$$k_{AB} = 0.333_{kN}(tensile\ )$$

Joint F :

$$\sum H = 0\ gives$$

k_{FA} cos 45° + k_{FE} = 0

k_{FA} cos 45° + k_{FE} = - k_{FA} cos 45° = -0.471*cos45°= - 0.333 kN

k_{FE} = 0.333kN (comp)

$\sum V = 0$ *gives*

k_{FA} cos 45° - k_{FB} = 0

k_{FA} cos 45° = k_{FB}

0.471 cos 45° = k_{FB}

k_{FB} = 0.333 kN (tensile)

Joint B:

$\sum V = 0$ *gives*

K_{BE} cos 45° + k_{FB} = 0

$$k_{BE} = \frac{-k_{BF}}{\cos 45°} = -\frac{0.333}{\cos 45°} = -0.471$$

$k_{BE}\, 0.471 \;\; kN\,(comp)$

$\sum H = 0$ *gives*

k_{BC} – k_{BA} + k_{BE} cos 45° = 0

k_{BC} = k_{BA} - k_{BE} cos 45°

= 0.333 – (-0.471) cos 45°

k_{BC} = 0.666 kN (tensile)

Joint C:

$\sum V = 0$ *gives*

K_{CE} – 1 = 0

K_{CE} + I kN (tensile)

$\sum H = 0$ *gives*

k_{CD} - k_{CB} = 0

k_{CD} = k_{CB} = 0.666

k_{CD} = 0.666 kN (tensile)

Joint D:

$\sum H = 0$ *gives*

k_{DE} cos45° + k_{DC} = 0

k_{DE} = - k_{DC} / cos45°

= -0.666 / cos 45°

= -0.942

k_{DE} = 0.942 kN (comp)

Real Forces F: The real forces in the members due to the given system of external loads are calculated using the method of joints.

By symmetry

$$R_A = R_D = \frac{Total\ Load}{2} = 50\ kN$$

Joint A: Initially assume all forces to be tensile.

$$\sum V = 0\ gives$$

$F_{AF} \cos 45° + 50 = 0$

$F_{AF} = -50 / \cos 45° = -70.71\ kN$

$F_{AF} = 70.71\ kN(comp)$

$\sum H = 0\ gives$

$F_{AF} \cos 45° + F_{AB} = 0$

$(-70.71) \cos 45° + F_{AB} = 0$

$F_{AB} = 50\ kN(tensile)$

Joint F :

$\sum H = 0\ gives$

$F_{FA} \cos 45^o + F_{FE} = 0$

$F_{FA} \cos 45^o + F_{FE} = - F_{FA} \cos 45^o = -70.71*\cos45^o = -50$ kN

$F_{FE} = -50$ kN (comp)

$$\sum V = 0\ gives$$

$F_{FA} \cos 45^o - F_{FB} = 0$

$F_{FA} \cos 45^o = F_{FB}$

$70.71 \cos 45^o = F_{FB}$

$F_{FB} = 50$ kN (tensile)

Joint B:

$\sum V = 0\ gives$

$F_{BE} \cos 45^o + F_{FB} - 50 = 0$

$F_{BE} \cos 45^o = - F_{BF} + 50 = - 50 + 50 = 0$

$F_{BE} = 0S$

$\sum H = 0\ gives$

$F_{BC} – F_{BA} + F_{BE} \cos 45^o = 0$

$F_{BC} + 0 - 50 = 0$

$F_{BC} = 50$ kN (tensile)

Joint C:

$\sum V = 0$ *gives*

$F_{CE} - 50 = 0$

$F_{CE} = 50$ kN (tensile)

$\sum H = 0$ *gives*

$F_{CD} - F_{CB} = 0$

$F_{CD} = F_{CB} = 50$

$F_{CD} = 50$ kN (tensile)

Joint D:

$\sum H = 0$ *gives*

$F_{DE} \cos 45^\circ + F_{DC} = 0$

$F_{DE} = - F_{DC} / \cos 45^\circ$

$= -50/ \cos 45^\circ$

$= -70.71$ kN

$F_{DE} = 70.71$ kN (comp)

Virtual – Work Equation:

$$\Delta = \sum \frac{kFL}{AE}$$

$$AF = BE = DE = \sqrt{3^2 + 3^2} = 4.243\ m$$

S.No	Member	k	F (kN)	L (m)	kFL (kN m)
1	AF	-0.471	-70.71	4.243	141.311
2	FE	-0.333	-50.00	3.00	49.950
3	ED	-0.942	-70.71	4.243	282.621
4	DC	0.666	50.00	3.00	99.900
5	CB	0.666	50.00	3.00	99.900
6	BA	0.333	50.00	3.00	49.950
7	FB	0.333	50.00	3.00	49.950
8	BE	-0.471	0.00	4.243	0.00
9	EC	1.000	50.00	3.00	150.00
				$\sum kFL =$	923.582

$$\sum kFL = 923.582 * 1000^2\ Nmm$$

$$(\Delta_C)_V = \sum \frac{kFL}{AE} = \frac{923.582 * 1000^2}{400 * 2 * 10^5} = 11.54\ mm$$

Vertical displacement of joint C = 11.54 mm (downward)

2. Using the principle of virtual work, determine the vertical and horizontal deflection components of joint C of the truss in fig. A = 150*10^{-6} m^2 and E = 200*10^6 kN/m^2

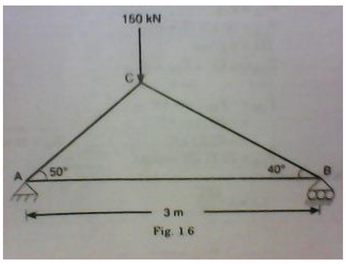


Fig. 1.6

Solution:

$$\Delta = \sum \frac{kFL}{AE}$$

Virtual Forces, k_h (for horizontal displacement at C)

Remove the real (external load) and apply a unit horizontal force at C of the truss.

$\sum H = 0$ *gives* , H_A = 1 kN

$(CC^1)/x = \tan 50^o$

$CC^1 = x \tan 50^o = (3-x) \tan 40^o$

$X = (3-x) \tan 40^o / \tan 50^o$

$X = 2.112 – 0.704x$

X = 1.239 m

(3-x) = 3.0-1.239 = 1.761 m

CC^1 = 1.239*tan 50º = 1.477 m

$AC = \sqrt{(1.239)^2 + (1.477)^2} = 1.928\ m$

$BC = \sqrt{(1.761)^2 + (1.477)^2} = 2.298\ m$

Taking moment about A,

V_B * 3 -1 * 1.477 = 0

V_B = 0.492 kN

V_A = 0.492 kN

Joint A:

$\sum V = 0$ *gives*

$K_{AC} \cos 40^o - 0.492 = 0$

K_{AC} = 0.492/cos40º = 0.642 kN (tensile)

$\sum H = 0$ *gives*

$k_{AC} \cos 50^o + k_{AB} – 1 = 0$

$0.642 \cos 50^o + k_{AB} - 1 = 0$

k_{AB} = 0.587 kN(tensile)

Joint C:

$\sum V = 0$ *gives*

$k_{CA} \cos 40^o + k_{CB} \cos 50^o = 0$

k_{CB} = 0.765 kN (comp)

Virtual forces k_v : (for vertical displacement at C)

Remove the real (external load) and apply a unit vertical force at C of the truss.

Taking moment about B,

V_A *3-1*1.761 = 0

V_A' = 0.587 kN

V_B' = 1-0.587 = 0.413 Kn

Joint A:

$\sum V = 0$ *gives*

K_{AC} cos40º +0.587 = 0

K_{AC} = -0.587/cos40º = -0.766 kN (comp)

$\sum H = 0$ *gives*

k_{AC} cos50º + k_{AB} = 0

k_{AB} = 0.492 kN(tensile)

Joint C:

$\sum V = 0$ *gives*

0.766 cos 40º - 1 - k_{CB} cos 50º = 0

k_{CB} = 0.643 kN (comp)

Real Forces F: The real forces in the members due to the given external load are calculated as below:

The only given force is of magnitude 150 kN and applied at C vertically. Therefore the forces in the members will be 150 times the k_V values.

F_{AB} = 0.492 * 150 = 73.8 kN (tensile)
F_{AC} = 0.766 * 150 = 114.9 kN (comp)
F_{CB} = 0.643 * 150 = 96.45 kN (comp)

Virtual – Work Equation:

$$\Delta = \sum \frac{kFL}{AE}$$

S.No	Member	k_H	K_V	F (kN)	L(m)	Fk_HL	Fk_VL
1	AB	0.587	0.492	73.8	3.0	129.962	108.929
2	AC	0.642	-0.766	-114.9	1.928	-142.22	169.693
3	CB	-0.765	-0.643	-96.45	2.298	169.556	142.516
					$\sum$	157.298	421.135

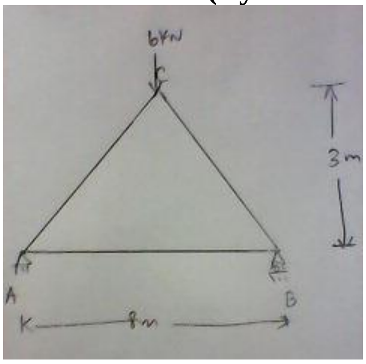
Horizontal deflection of point C = $(\Delta_C)_h = \sum \frac{k_h FL}{AE}$

$= 157.298 / (200 * 10^6 * 150 * 10^{-6}) = 0.00524$ m = 5.24 mm

Vertical deflection of point C = $(\Delta_C)_v = \sum \frac{k_v FL}{AE}$

$= 421.135 / (200 * 10^6 * 150 * 10^{-6}) = 0.14$ m = 14 mm

3. Determine the vertical and horizontal displacements of the point C of the pin-jointed frame shown in fig. The cross sectional area of AB is 100 sqmm and of AC and BC 150 mm^2 each. E= 2 x 10^5 N/mm^2. (By unit load method)



Sol:

The vertical and horizontal deflections of the joint C are given by

$$\delta_V = \sum \frac{PuL}{AE}$$

$$\delta_H = \sum \frac{Pu'L}{AE}$$

A) Stresses due to External Loading:

AC = $\sqrt{3^2 + 4^2} = 5m$

Reaction:

R_A = -3/4

R_B = 3/4

Sin θ = 3/5 = 0.6; Cos θ = 4/5 = 0.8

Resolving vertically at the joint C, we get

$6 = P_{AC} \cos\theta + P_{BC} \sin\theta$

Resolving horizontally at the joint C, we get

$P_{AC} \cos\theta = P_{BC} \sin\theta$; P_{AC} =

$P_{BC}\ P_{AC} \sin\theta + P_{BC} \sin\theta = 6$

$2 P_{AC} \sin\theta = 6$

$P_{AC} = 6/\sin\theta = 6/2 \times 0.6 = 5$ KN (tension)

$P_{AC} = P_{BC} = 5$ KN (tension)

Resolving horizontally at the joint C, we get

$P_{AB} = P_{AC} \cos\theta$

$P_{AB} = 5 \cos\theta$; $P_{AB} = 5 \times 0.8$

$P_{AB} = 4$ KN (comp)

B) Stresses due to unit vertical load at C:

Apply unit vertical load at C. The Stresses in each member will be 1/6 than of those obtained due to external load.

$$u_{AC} = u_{BC} = 5/6$$

$$u_{AB} = -4/6 = -2/3$$

C) Stresses due to unit horizontal load at C:

Assume the horizontal load towards left as shown in fig.

Resolving vertically at the joint C, we get

$$(u_{CA})'\sin\theta = (u_{CB})'\sin\theta$$

$\therefore$

$$(u_{CA})' = (u_{CB})'$$

Resolving horizontally at the joint C, we get

$$(u_{CB})'\cos\theta + (u_{CA})'\cos\theta = 1$$

$$(u_{CB})'\cos\theta + (u_{CB})'\cos\theta = 1$$

$$2u_{CB}'\cos\theta = 1$$

$$u_{CB}' = \frac{1}{2\cos\theta} = \frac{1}{2x0.8} = 5/8KN\,(tension)$$

$$\therefore\ u_{CA}' = -5/8KN$$

$$u_{CA}' = 5/8KN\,(comp)$$

Resolving horizontally at the joint B, we get

$$u_{AB}' = -u_{BC}'\cos\theta$$

$$u_{AB}' = -5/8x0.8 = -0.5KN$$

$$u_{AB}' = 0.5KN\,(comp)$$

Member	Length(L) mm	Area $(mm)^2$	P(KN)	U (kN)	PUL/A	U'(KN)	PU'L/A
AB	8000	100	-4	-2/3	640/3	-1/2	160
BC	5000	150	5	5/6	2500/18	5/8	2500/24
CA	5000	150	5	5/6	2500/18	-5/8	2500/24

E = 2 X 10^5 n/mm^2= 200 KN/m^2

$$\delta v = \sum \frac{Pul}{AE} = \frac{491}{200} = 2.45mm$$

$$\delta h = \sum \frac{pu'l}{AE} = \frac{160}{200} = 0.8mm$$

4. Using the principle of least work, analyze the portal frame shown in Fig.

Sol:

The support is hinged. Since there are two equations at each supports. They are H_A, V_A, H_D, and V_D. The available equilibrium equation is three. (i.e.) $\sum M = 0, \sum H = 0, \sum V = 0$.

$\therefore$ The structure is statically indeterminate to first degree. Let us treat the horizontal H ($\leftarrow$) at A as redundant. The horizontal reaction at D will evidently be = (3-H) ($\leftarrow$).

By taking moments at D, we get

$(V_A \times 3) + H(3-2) + (3 \times 1)(2-1.5) - (6 \times 2) = 0$

$$V_A = 3.5 - H/3$$

$$V_D = 6 - V_A = 2.5 + H/3$$

By the theorem of minimum strain energy,

$$\frac{\partial U}{\partial H} = 0$$

$$\frac{\partial U_{AB}}{\partial H} + \frac{\partial U_{BE}}{\partial H} + \frac{\partial U_{CE}}{\partial H} + \frac{\partial U_{DC}}{\partial H} = 0$$

(1) For member AB:

Taking A as the origin.

$$M = \frac{-1.x^2}{2} + H.x$$

$$\frac{\partial M}{\partial H} = x$$

$$\frac{\partial U_{AB}}{\partial H} = \frac{1}{EI}\int_0^3 M\frac{\partial M}{\partial H}dx$$

$$= \frac{1}{EI}\int_0^3 \left(\frac{-x^2}{2} + Hx\right)x\ dx$$

$$= \frac{1}{EI}\left[\frac{Hx^3}{3} - \frac{x^4}{8}\right]_0^3$$

$$= \frac{1}{EI}\left[9H - 10.12\right]$$

(2) For the member BE:

Taking B as the origin.

$$M = (H \; x \; 3) - (3 \; x \; 1 \; 1.5) + \left(3.5 \quad \frac{H}{3}\right)x$$

$$M = 3H - 4.5 + 3.5x - \frac{Hx}{3}$$

$$\frac{\partial M}{\partial H} = 3 - \frac{x}{3}$$

$$\frac{\partial U_{BE}}{\partial H} = \frac{1}{EI}\int_0^1 M \frac{\partial M}{\partial H} dx$$

$$= \frac{1}{EI}\int_0^1 \left(3H - 4.5 + 3.5x - \frac{Hx}{3}\right)\left(3 - \frac{x}{3}\right)dx$$

$$= \frac{1}{EI}\int_0^1 \left(9H - 13.5 + 10.5x - Hx - Hx + 1.5x - 1.67x^2 + \frac{Hx^2}{9}\right)dx$$

$$= \frac{1}{EI}\int_0^1 \left(9H - 13.5 + 12x - 2Hx - 1.67x^2 + \frac{Hx^2}{9}\right)dx$$

$$= \frac{1}{EI}\left(9Hx - 13.5x + 6x^2 - Hx^2 - 0.389x^3 + \frac{Hx^3}{27}\right)_0^1$$

$$= \frac{1}{EI}\left(9H - 13.5 + 6^2 - H - 0.389 + \frac{H}{27}\right)$$

$$= \frac{1}{EI}[9H - 7.9]$$

(3) For the member CE:

Taking C as the origin

$$M = -(3 - H)x2 + (2.5 + \frac{H}{3})x$$

$$M = -6 + 2H + 2.5x + \frac{Hx^3}{3}$$

$$\frac{\partial U_{CE}}{\partial H} = \frac{1}{EI}\int_0^2 M \frac{\partial M}{\partial H}$$

$$= \frac{1}{EI}\int_0^2 \left[\left(-6 + 2H + 2.5x + \frac{Hx}{3}\right)\left(2 + \frac{x}{3}\right)\right]$$

$$= \frac{1}{EI}\int_0^2 \left[-12+4H+5x+6.67Hx-2x+6.67Hx+0.833x^2+\frac{Hx^2}{9}\right]dx$$

$$= \frac{1}{EI}\int_0^2 \left[-12+4H+3x+13.34Hx-2x+0.833x^2+\frac{Hx^2}{9}\right]dx$$

$$= \frac{1}{EI}(10.96H - 15.78)$$

(4) For the member DC:

Taking D as the origin

$$M = -(3-H)x = -3x + Hx$$

$$\frac{\partial M}{\partial x} = x$$

$$\frac{\partial U_{DC}}{\partial H} = \frac{1}{EI}\int_0^2 M\frac{\partial M}{\partial H}dx$$

$$= \frac{1}{EI}\int_0^2 (-3x+Hx)(x)\,dx \qquad = \frac{1}{EI}\int_0^2 \left(-3x^2+Hx^2\right)dx$$

$$= \frac{1}{EI}\left(\frac{-3x^3}{3}+\frac{Hx^3}{3}\right)_0^2 dx \qquad = \frac{1}{EI}\left(-x^3+\frac{Hx^3}{3}\right)_0^2 dx$$

$$= \frac{1}{EI}(2.67H - 8)$$

Subs the values

$$\frac{\partial U}{\partial H} = 0$$

1/EI (9-10.2) + (8.04H-7.9) + (10.96H-15.78) + (-8+2.67H) = 0

30.67H = 41.80

H = 1.36 KN

Hence

V_A = 3.5 - H/3 = 3.5 - 1.36/3 = 3.05 KN

V_D = 2.5 + H/3 = 2.5 + 1.36/3 = 2.95 KN

$M_A = M_D$
=0

M_B = (-1 x 3^2)/2 + (1.36 x 3) = -0.42 KN –m

M_C = - (3-H) 2 = - (3-1.36)2 =-3.28KNm

5. A simply supported beam of span 6m is subjected to a concentrated load of 45 KN at 2m from the left support. Calculate the deflection under the load point. Take E = 200 x 10^6 KN/m^2 and I = 14 x 10^{-6} m^4.

Solution:

Taking moments about B.

V_A x 6 – 45 x 4=0

V_A x 6 -180 = 0

V_A = 30 KN

V_B = Total Load – V_A = 15 KN

Virtual work equation:

$$(\delta_c)_V = \int_0^L \frac{mMdx}{EI}$$

Apply unit vertical load at c instead of 45 KN

R_A x 6-1 x 4 =0

R_A = 2/3 KN

R_B = Total load –R_A = 1/3 KN

Virtual Moment:

Consider section between AC

M_1 = 2/3 X_1 [limit 0 to 2]

Section between CB

M_2 = 2/3 X_2-1 (X_2-2) [limit 2 to 6]

Real Moment:

The internal moment due to given loading

M_1= 30 x X_1

M_2 = 30 x X_2 -45 (X_2 -2)

$$(\delta_c)_V = \int_0^2 \frac{m_1 M_1 dx_1}{EI} + \int_2^6 \frac{m_2 M_2 dx_2}{EI}$$

$$= \int_0^2 \frac{\left(\frac{2x_1}{3}\right)(30x_1)}{EI}dx_1 + \int_2^6 \frac{\left(\frac{2}{3}x_2 - (x_2 - 2)\right)(30x_2 - 45(x_2 - 2))}{EI}dx_2$$

$$= \frac{1}{EI}\int_0^2 20x_1^2 + \int_2^6 \left(\frac{2}{3}x_2 - x_2 + 2\right)(30x_2 - 45x_2 + 90)dx_2$$

$$= \frac{1}{EI}\int_0^2 20x_1^2 + \int_2^6 \left(\frac{-x_2}{3} + 2\right)(-15x_2 + 90)dx_2$$

$$= \frac{1}{EI}\int_0^2 20x_1^2 + \int_2^6 5x_2^2 - 30x_2 - 30x_2 + 180dx_2$$

$$= \frac{1}{EI}\left[\frac{20x_1^3}{3}\right]_0^2 + \left[\frac{5x_2^3}{3} - \frac{60x_2^3}{2} + 180x_2\right]_2^6$$

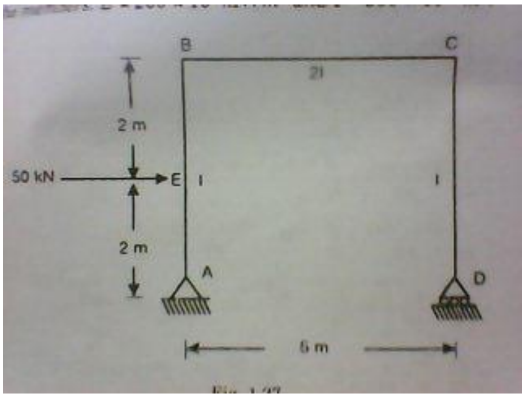
$$= \frac{20}{EI}\left(\frac{8}{3}\right) + \frac{1}{EI}\left(\frac{5}{3}(6^3 - 2^3) - 30(6^2 - 2^2) + 180(6 - 21)\right)$$

$$= \frac{1}{EI}[53.33 + 346.67 - 960 + 720]$$

$$= \frac{160}{EI} = \frac{160}{200x10^6 x14x10^{-6}} = 0.0571\ m\ (or)\ 57.1\ mm$$

The deflection under the load = 57.1 mm

6. Using the method of virtual work, determine the horizontal displacement of support D of the frame shown in fig. The values of I are indicated along the members. Take E = 200 x 10^6 KN/m^2 and I = 300 x 10^{-6} m^4.



Solution:

$$\Delta = \int_0^l m\frac{M}{EI}dx$$

Virtual moments, m. Remove the external load and apply a unit load in the horizontal direction at d. the support reactions and internal virtual moments are computed as under.

(Sign for moments: Left clockwise +ve: Right clockwise +ve)

$m_1 = 1.x_1$ limits 0 to 2 m

$m_2 = 1.(2+x_2)$ limits 0 to 2 m

$m_3 = 1.x_3$ limits 0 to 4 m

$m_4 = 1X4$ limits 0 to 5 m

Real moments, M. Due to the given loading, the support reactions and real moments are computed as under.

$\sum H = 0$ gives $H_A = 50$ kN (←)

Taking moments about A,

V_D x 5 – 50 x 2 = 0

V_D = 50 x 2/5 = 20 kN (↑)

$\sum V = 0$ gives

V_A = 20 kN (↓)

$M_1 = 50 X x_1$

$M_2 = 50 X (2 + x_2) – 50 X x_2 = 100$ kNm

$M_3 = 0$

$M_4 = 20 X x_4$

Virtual Work Equation:

$$(\Delta_D)_h = \int\frac{mMdx}{EI}$$

$$= \int_0^2\frac{(1.x_1)(50x_1)dx_1}{EI} + \int_0^2\frac{1(2+x_2)(100)dx_2}{Ei} + \int_0^4\frac{(1*x_3)*0dx_3}{EI} + \int_0^5\frac{(1*4)(20*x_4)dx_4}{EI}$$

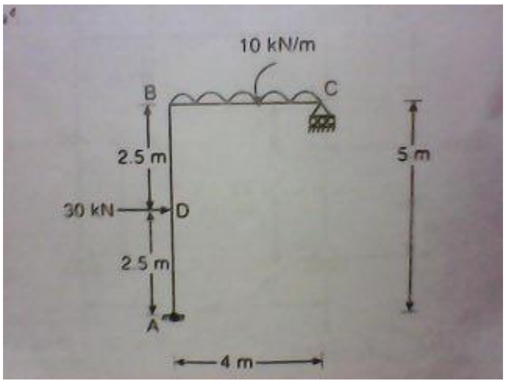
$$= \frac{1}{EI}\left\{\left[50\frac{x_1^3}{3}\right]_0^2 + \left[\frac{200x_2 + 100x_2^2}{2}\right]_0^2 + \left[\frac{40x_4^2}{2}\right]_0^5\right\}$$

= 1/EI (1333.33 +600 +500) = 1233.33/EI

$=1233.33/ 200*10^6 *300 *10^{-6} = 0.02056\text{m} = 20.56$ mm

Horizontal displacement of support D, $(\Delta_D)_h$ = 20.56 mm (→)

7. Using the method of virtual work, determine the horizontal displacement of support D of the frame shown in fig. The values of I are indicated along the members. Take E = 200 x 10^6 KN/m^2 and I = 4 x 10^{-6} m^4.



Solution:

$$\Delta = \int_0^l \frac{mMdx}{EI}$$

Virtual moments, m. Remove the external load and apply unit horizontal load at C. The support reactions and internal virtual moments are computed as shown.

$\sum H = 0$ gives $\quad H_A = 1$ kN (←)

Taking moments about C, $\quad V_A *4 + 1* 5 = 0$

$4V_A = -5$

$V_A = -5/4 = -12.5 \quad$ i.e. $V_A = 1.25$ kN (↓)

$\sum V = 0$ gives

$V_C = 1.25$ kN (↑)

$M_1 = 1.x_1$ (x_1 varies from 0 to 2.5 m)

$M_2 = 1.x_2$ (x_2 varies from 2.5 to 5.0 m)

$M_3 = 1.25x_3$(x_3 varies from 0 to 4 m)

Real moments. Due to the given external loading, the support react ions and real moments are computed as shown below.

$\sum H = 0$ gives $\quad H_A = 30$ kN (←)

Taking moments about A, $\quad R_A *4 - 10* 4 * 4/2 – 30 * 2.5 = 0$

$4R_C = 80+75 = 155$

$R_C = 38.75$ kN

R_A = Total load – R_C = 10*4 – 38.75 = 1.25 kN

$M_1 = = 30*x_1$ (x_1 varies from 0 to 2.5)

$M_2 = 30x_2 – 30(x_2 – 2.5)$ $\quad$ (x_2 varies from 2.5 to 5)

Virtual Work Equation:

$$(\Delta_C)_h = \int \frac{mMdx}{EI}$$

$$= \int_0^{2.5} \frac{(1.x_1)(30x_1)dx_1}{EI} + \int \frac{(1.x_2)\left[30x_2 - 30(x_2 - 2.5)\right]}{EI} dx_2$$

$$+ \int_0^4 \frac{(1.25x_3)(38.75x_3 - 5x_3^{\,2})dx_3}{EI}$$

$$= \frac{30}{EI}\left[\frac{x_1^{\,3}}{3}\right]_0^{2.5} - \frac{75}{EI}\left[\frac{x_2^{\,2}}{2}\right]_{2.5}^{5.0} + \frac{1}{EI}\left[48.44\frac{x^3}{3} - 6.25\frac{x^4}{4}\right]_0^4$$

$$= \frac{1}{EI}\left[156.26 - \frac{75}{2}\left(5^2 - 2.5^2\right) + \frac{48.44}{3}x4^3 - \frac{6.25}{4}x4^4\right]$$

$$(\Delta_C)_h = \frac{86.512}{2 * 10^8 * 4 * 10^{-6}} = 0.108\ m = 108\ mm$$

Horizontal displacement of point C = 108 mm

CE6501- STRUCTURAL ANALYSIS –CLASSICAL METHODS
(FOR V – SEMESTER)

UNIT – III
(ARCHES)

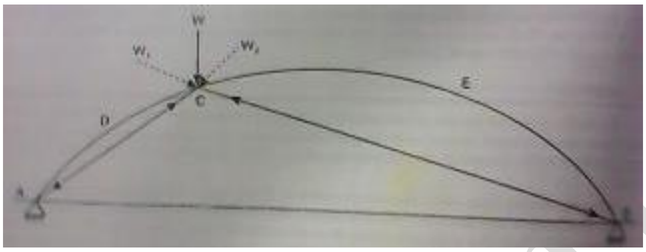
ARCH

An arch is defined as a curved girder, having convexity upwards and supported at its ends.

The supports must effectively arrest displacements in the vertical and horizontal directions. Only then there will be arch action.

Linear arch

If an arch is to take loads, say W_1, W_2, and W_3 and a vector diagram and funicular polygon are plotted as shown; the funicular polygon is known as the linear arch or theoretical arch.



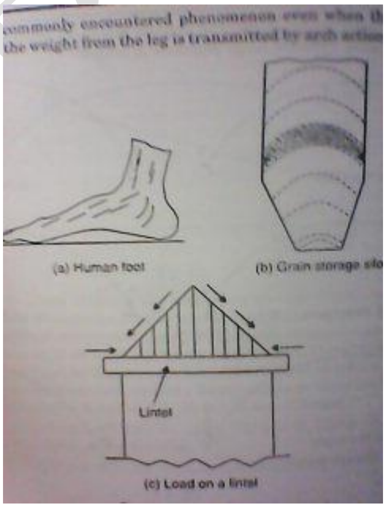
The polar distance 'ot' represents the horizontal thrust.

The links AC, CD, DE and EB will br under compression and there will be no bending moment. If an arch of this shape ACDEB is provided, there will be no bending moment.

Eddy's theorem.

Eddy's theorem states that "The bending moment at any section of an arch is proportional to the vertical intercept between the linear arch (or theoretical arch) and the center line of the actual arch".

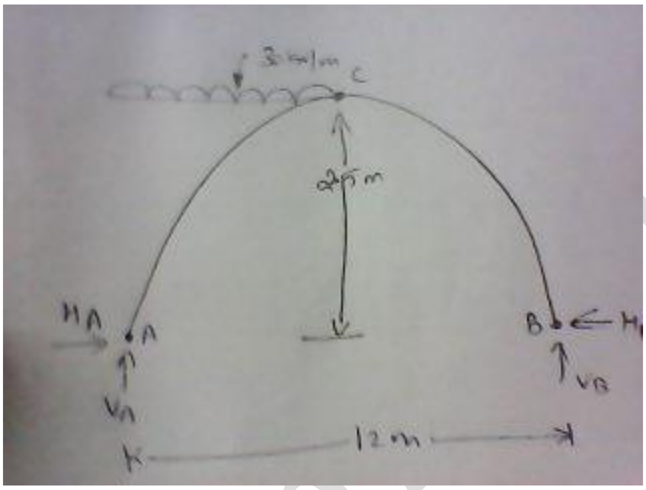
$$BM_x = \text{ordinate } O_2\,O_3 * \text{scale factor}$$



Degree of static indeterminacy of a three hinged parabolic arch

For a three-hinged parabolic arch, the degree of static indeterminacy is zero. It is statically determinate.

A three hinged parabolic arch hinged at the crown and springing has a horizontal span of 12m and a central rise of 2.5m. it carries a udl of 30 kN/m run over the left hand half of the span. Calculate the resultant at the end hinges.



Let us take a section X of an arch. Let θ be the inclination of the tangent at X. if H is the horizontal thrust and V the net vertical shear at X, from theb free body of the RHS of the arch, it is clear that V and H will have normal and radial components given by,

$N = H \cos\theta + V \sin\theta$

$R = V \cos\theta - H \sin\theta$

The normal thrust and radial shear in an arch rib.

Parabolic arches are preferable to carry distributed loads. Because, both, the shape of the arch and the shape of the bending moment diagram are parabolic. Hence the intercept between the theoretical arch and actual arch is zero everywhere. Hence, the bending moment at every section of the arch will be zero. The arch will be under pure compression that will be economical.

Difference between the basic action of an arch and a suspension cable

An arch is essentially a compression member, which can also take bending moments and shears. Bending moment and shears will be absent if the arch is parabolic and the loading uniformly distributed.

A cable can take only tension. A suspension bridge will therefore have a cable and a stiffening girder. The girder will take the bending moment and shears in the bridge and the cable, only tension.

Because of the thrust in cables and arches, the bending moments are considerably reduced.

If the load on the girder in uniform. The bridge will have only cable tension and no bending moment on the girder.

Under what conditions will the bending moment in an arch be zero throughout

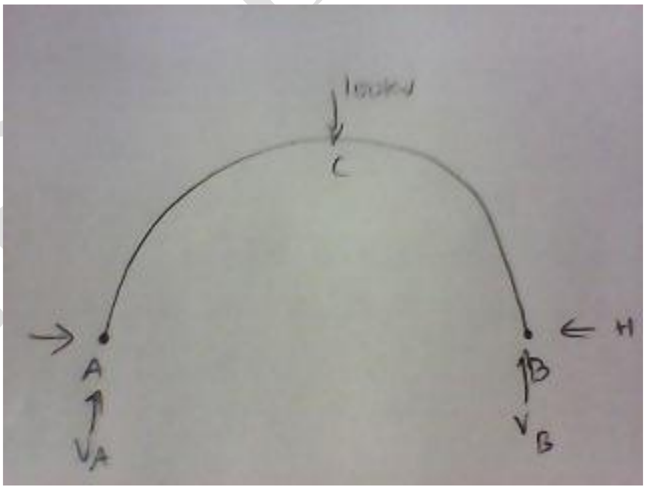
The bending moment in an arch throughout the span will be zero, if

(i) The arch is parabolic and

(ii) The arch carries udl throughout the span

A three-hinged semicircular arch carries a point load of 100 kN at the crown. The radius of the arch is 4m. Find the horizontal reactions at the supports.

$V_A = V_B = 50$ kN

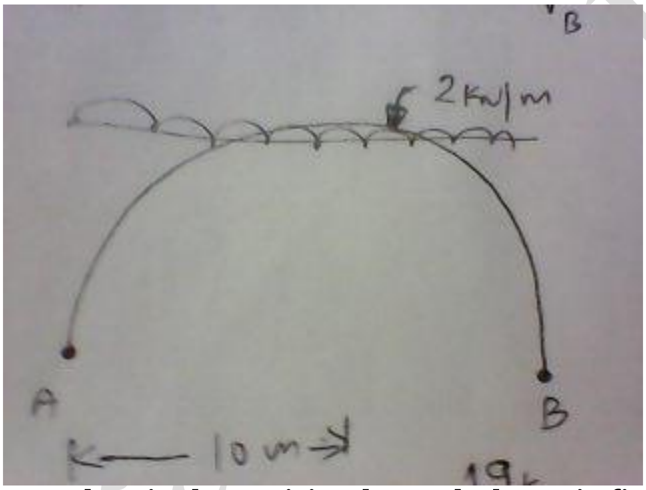


Equating the moment about C to Zero, $V_A * 4 - H*4 = 0$

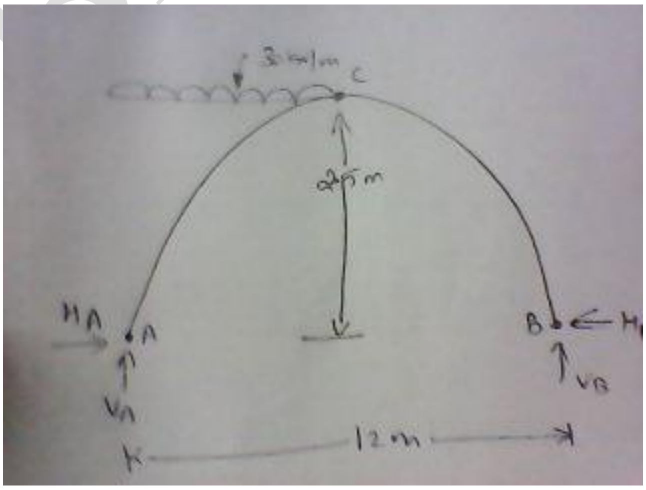
$$H = V_A$$

Horizontal reaction, H = 50 kN

A three-hinged semicircular arch of radius 10m carries a udl of 2 kN/m over the span. Determine the horizontal and vertical reactions at the supports.



Determine H, V_A and V_B in the semicircular arch shown in fig



Equating moments about A to Zero,

$V_B * 12 - 12 * 9 = 0;$

$V_B = 9$ kN and $V_A = 3$ kN

Equating moments to the left of C to zero,

$H = V_A = 3$ kN; H= 3 kN

Distinguish between two hinged and three hinged arches.

SI.NO	Two hinged arches	Three hinged arches
1.	Statically indeterminate to first degree	Statically determinate
2.	Might develop temperature stresses.	Increase in temperature causes increases in central rise. No stresses
3.	Structurally more efficient.	Easy to analyse. But, in construction, the central hinge may involve additional expenditure.
4.	Will develop stresses due to sinking of supports	Since this is determinate, no stresses due to support sinking

Rib – shorting in the case of arches.

In a 2-hinged arch, the normal thrust, which is a compressive force along the axis of the arch, will shorten then rib of the arch. This is turn will release part of the horizontal thrust.

Normally, this effect is not considered in the analysis (in the case of two hinged arches). Depending upon the important of the work we can either take into account or omit the effect of rib shortening. This will be done by considering (or omitting) strain energy due to axial compression along with the strain energy due to bending in evaluating H.

Effect of yielding of support in the case of an arch.

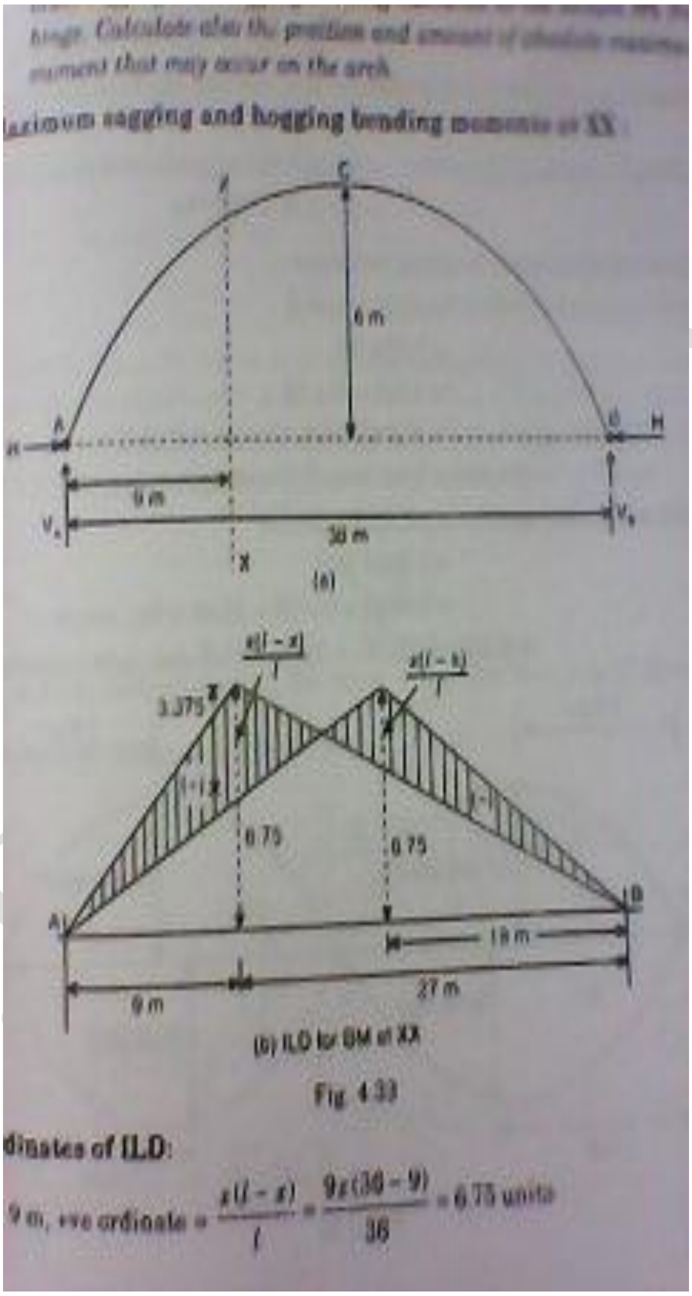
Yielding of supports has no effect in the case of a 3 hinged arch which is determinate. These displacements must be taken into account when we analyse 2 hinged or fixed arches as under

$$\frac{\partial U}{\partial H} = \Delta H \text{ instead of zero}$$

$$\frac{\partial U}{\partial V_A} = \Delta V_A \text{ instead of zero}$$

Here U is the strain energy of the arch ΔH and ΔV_A are the displacements due to yielding of supports.

A three-hinged parabolic arch has a horizontal span of 36m with a central rise of 6m. A point load of 40 kN moves across the span from the left to the right. What is the absolute maximum positive bending moment that wills occur in the arch



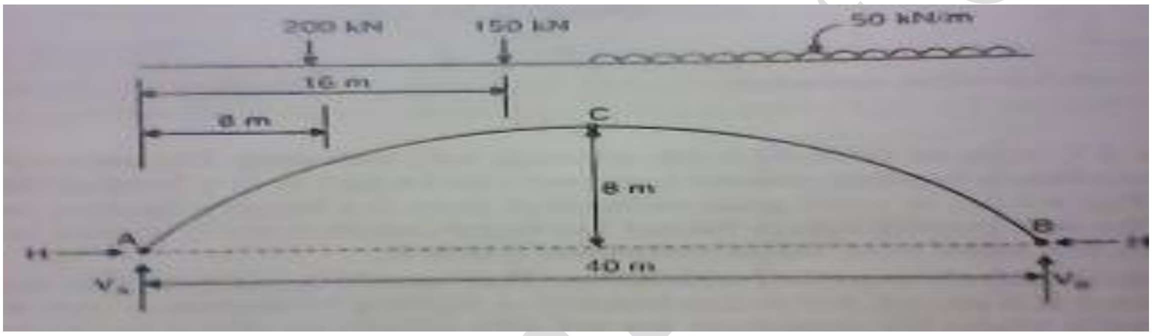
For a single concentrated load moving from one end to the other, Absolute maximum positive bending moment

= 0.096wl = 0.096*40 * 36=138.24 kNm

This occurs at 0.211 l = 0.211 * 36 = 7.596 m from the ends.

Absolute maximum positive bending moment = 138.24 kNm at 7.596 m from the ends.

A 3 hinged arch of span 40m and rise 8m carries concentrated loads of 200 kN and 150 kN at a distance of 8m and 16m from the left end and an udl of 50 kN/m on the right half of the span. Find the horizontal thrust.



Solution:

(a) Vertical reactions V_A and V_B :

Taking moments about A,

200(8) + 150(16) + 50 * 20 * (20 + 20/2) – V_B (40) = 0

1600 + 2400 + 30000 – 40 V_B = 0

V_B= = 850 kN

V_A = Total load – V_B = 200 + 150 + 50 * 20 – 850 = 500 kN

(b) Horizontal thrust (H)

Taking moments about C,

-H x 8 + V_A (20) – 200 (20 – 8) – 150 (20 – 16) = 0

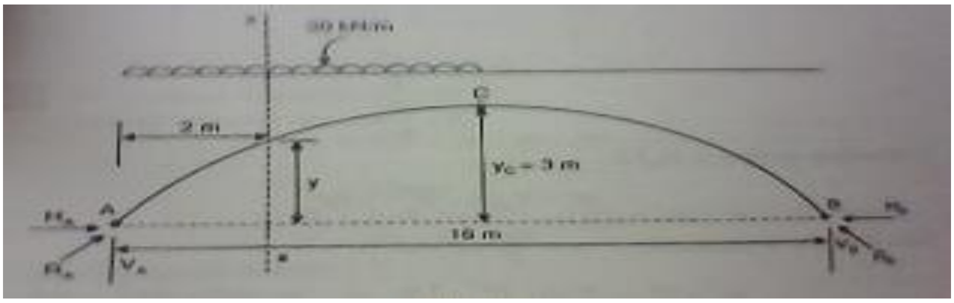
-8H + 500 * 20 – 200 (12) – 150 (4) = 0

H =

875 kN

A parabolic 3-hinged arch carries a udl of 30kN/m on the left half of the span. It has a span of 16m and central rise of 3m. Determine the resultant reaction at supports. Find the bending moment, normal thrust and radial shear at xx, and 2m from left support.

Solution:



(1) Reaction at A nd B;

(i) Vertical components of reactions;

Taking moments about A,

$$-V_B (16) + 30 \times 8^2 /2 = 0$$
$$- V_B (16) + 30 * 32 = 0$$
$$V_B = 60 \text{ kN}$$
$$V_A = \text{Total load} - V_B = 30 * 8 - 60 \text{ kN}$$
$$V_A = 180 \text{ kN}$$

(ii) Horizontal components of reactions at A and

Taking moments about the crown point C,

$$V_A * 8 - 30 * 8 * 8/2 - H_A * y_c = 0$$
$$180 * 8 - 30 * 32 = H_A * 3$$
$$H_A = 160 \text{ kN}$$
$$H_B = H_A = \text{since } \sum H = 0$$
$$H_B = 160 \text{ kN}$$

(iii) Resultants reactions at A and B;

$$R_A = \sqrt{V_A^2 + H_A^2} = \sqrt{(180)^2 + (160)^2} = 240.83 \ kN$$

$$R_B = \sqrt{V_B^2 + H_B^2} = \sqrt{(60)^2 + (160)^2} = 170.88 \ kN$$

(2) Bending moment at x = 2m from A:

Bending moment = $V_A (2) - 30 * 2 * 1 - H_A(y)$ -----------------(1)

Where, y = Rise of the arch at x = 2m from 'A':

For parabolic arches, $y = \frac{4r}{l^2} * x(l - x)$ at a distance of 'x' from thje support

Where, r = rise of the arch at Crown Point = 3m

$$y = \frac{4 * 3}{(16)^2} * 2(16 - 2)$$

Substitute in (1)y = 1.3125 m at x = 2m from 'A'.
Bending moment at x = 2m from A = 180 (2) – 30 * 2 * 1 – 160 * 1.3125
Bending moment at xx = 90 kNm

(3) Radial shear force at x = 2m from A

Shear force, $R_X = V_x \cos\theta - H \sin\theta$
Where, V = Net vertical shear force at x = 2m from A
$= V_A - w(2) = 180 - 30 * 2$
V = 120 kN

H = Horizontal shear force = 160 kN

$$\theta = \tan^{-1}\left[\frac{4r}{l^2}(l-2x)\right]$$

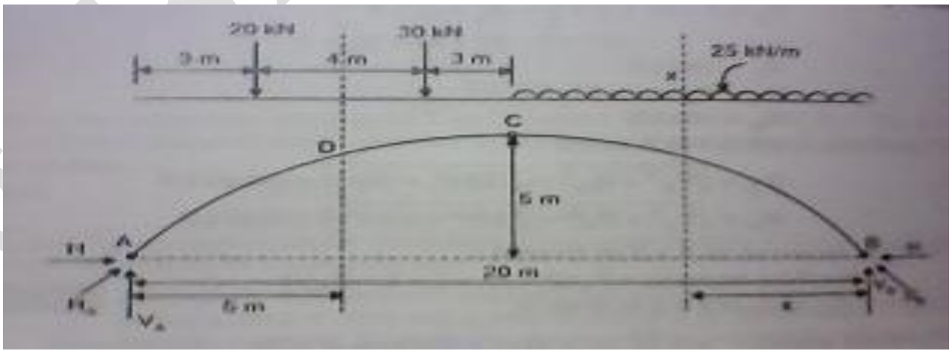
$$\theta = \tan^{-1}\left[\frac{4 * 3}{(16)^2}(16-2(2))\right]$$

θ = 29º21'
R = 120 cos 29º21' – 160 sin 29º21'
R = 26.15 kN

(4) Normal thrust at x = 2m from A:
Normal thrust $P_N = V_x \sin\theta + H\cos\theta$ = 120 sin 29º21' + 160 cos 29º21'
P_N = 198.28 kN.

A parabolic 3-hinged arch carries loads as shown in fig. Determine the resultant reactions at supports. Find the bending moment, normal thrust and radial shear at D, 5m from A. What is the maximum bending moment

Solution:



(1) Reaction at supports: (R_A and R_B)

(i) Vertical components of R_A and R_B : (V_A and V_B)
Taking moments about A,
20 * 3 + 30 (7) + 25 * 10 * (10 +10/2) – V_B *(20) = 0
V_B = 201 kN
V_A = Total load – V_B = 20 + 30 + 25 * 10 -201
V_A = 99 kN

(ii) Horizontal thrust (H):

Taking moments about the crown point C, considering the right side of ‘C’,

$-V_B\,(20/2) + H\,(5) + 25 * 10 * 5 = 0$

$-201 * (20/2) + 5\,H + 1250 + 0$

$H = 125$ kN

(iii) Resultant reactions R_A and R_B ;

$$\theta_A = \tan^{-1}\frac{V_A}{H} = \tan^{-1}\frac{99}{152} = 32°4'30''.6$$

$R_A = \sqrt{H^2 + V^2{}_A} = \sqrt{(152)^2 + (99)^2} = 181.39\ kN$

$R_B = \sqrt{H^2 + V^2{}_B} = \sqrt{(152)^2 + (201)^2} = 252\ kN$

$\theta_B = \tan^{-1} \frac{V_B}{H} = \tan^{-1} \frac{201}{152} = 52°54'9''.86$

2. Bending moment, normal thrust and radial shear force (at D):

(i) $y_D = \frac{4r}{l^2} x(l-x) = \frac{4*5}{(20)^2} * 5(20-5)$

$y_D = 3.75$ m

BMD = V_A (5) – Hy_D – 20 (5-3) = 99(5) – $152y_D$ – 20(2)

= 495 – 152 (3.75) -40

BMD = -115 kNm

(ii) Slope of the arch at D,

$$\tan\theta = \left(\frac{dy}{dx}\right)_D$$

$$\theta = \tan^{-1}\left(\frac{4r}{l^2}(l-2x)\right)$$

$$\theta = \tan^{-1}\left(\frac{4*5}{(20)^2}(20-2*5)\right)$$

$\theta = 26°33'55''.18$

(iii) Normal thrust

$P = V \sin\theta + H \cos\theta$

V = Net beam shear force = V_A – 20

V = 99 – 20 = 79 kN

Substitute in (iii) $P = 79 \sin\theta + 152 \cos\theta = 179.28$ kN

(iv) Radial shear force

$F = V \cos\theta - H \sin\theta$

$F = 79 \cos\theta - 152 \sin\theta = 2.683$ kN

3. Maximum Bending Moment in CB:

Considering a section xx at a distance of 'x' m from 'B'

BM_{xx} = 254KNM

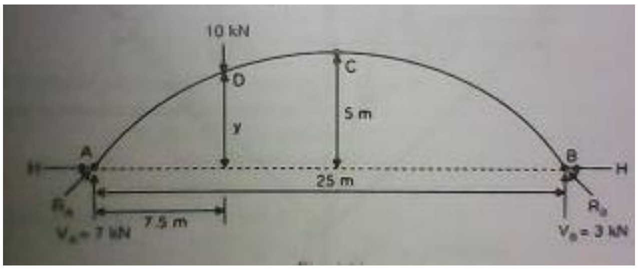
A 3-hinged arch is circular, 25 m in span with a central rise of 5m. It is loaded with a concentrated load of 10 kN at 7.5m from the left hand hinge. Find the

(a) Horizontal thrust

(b) Reaction at each end hinge

(c) Bending moment under the load

Solution:



Vertical reactions V_A and V_B:

Taking moments about A,

$10(7.5) - V_B(25) = 0$

$V_B = 3$ kN

$V_A = \text{Total load} - V_B = 10 - 3 = 7$ kN

1. Horizontal thrust (H):

Taking moments about C

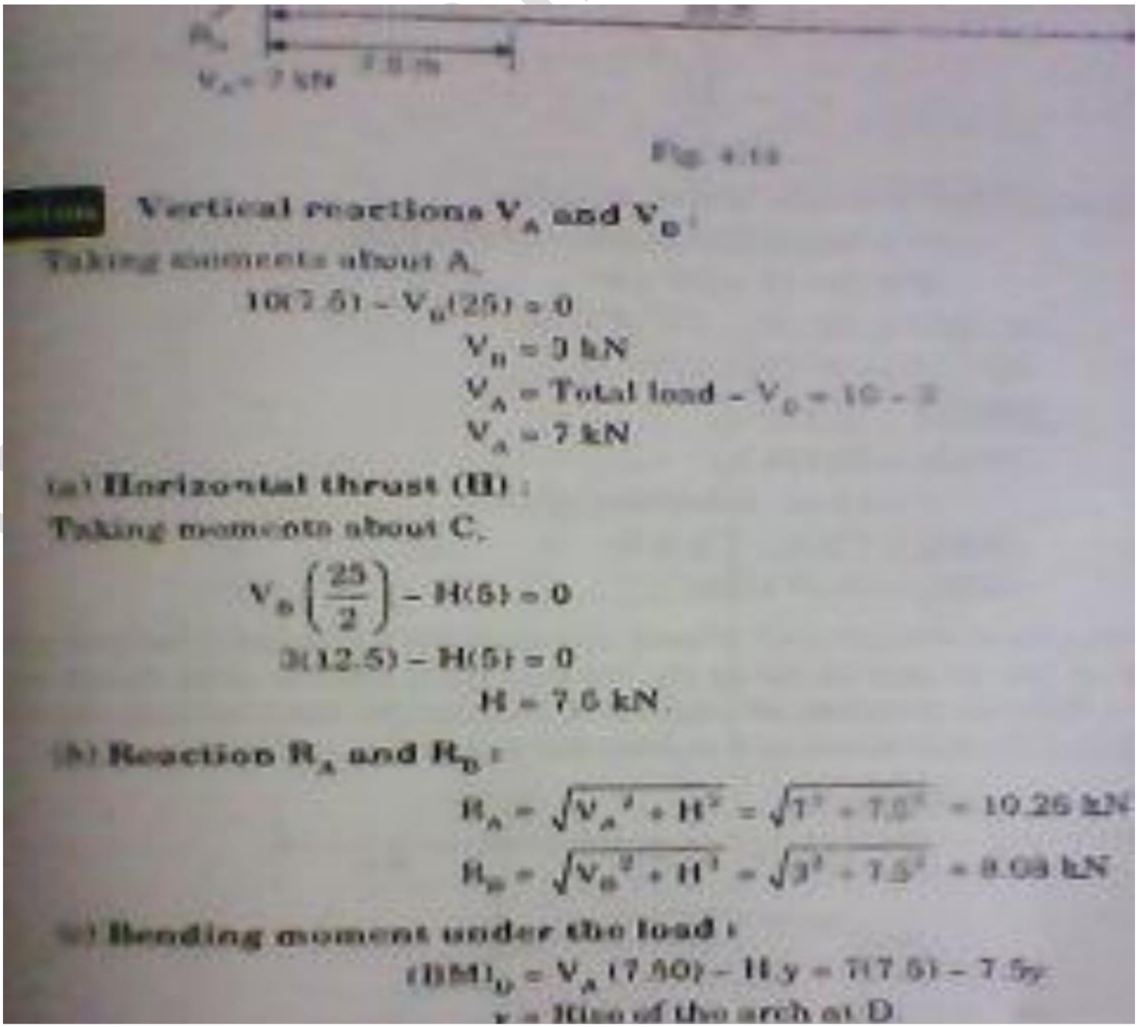


Fig. 4.14

Vertical reactions V_A and V_B:

Taking moments about A,

$10(7.5) - V_B(25) = 0$

$V_B = 3$ kN

$V_A = \text{Total load} - V_B = 10 - 3$

$V_A = 7$ kN

(a) Horizontal thrust (H):

Taking moments about C,

$$V_B\left(\frac{25}{2}\right) - H(5) = 0$$

$3(12.5) - H(5) = 0$

$H = 7.5$ kN

(b) Reaction R_A and R_B:

$$R_A = \sqrt{V_A^2 + H^2} = \sqrt{7^2 + 7.5^2} = 10.26 \text{ kN}$$

$$R_B = \sqrt{V_B^2 + H^2} = \sqrt{3^2 + 7.5^2} = 8.08 \text{ kN}$$

(c) Bending moment under the load:

$(BM)_D = V_A(7.50) - H.y = 7(7.5) - 7.5y$

y = Rise of the arch at D

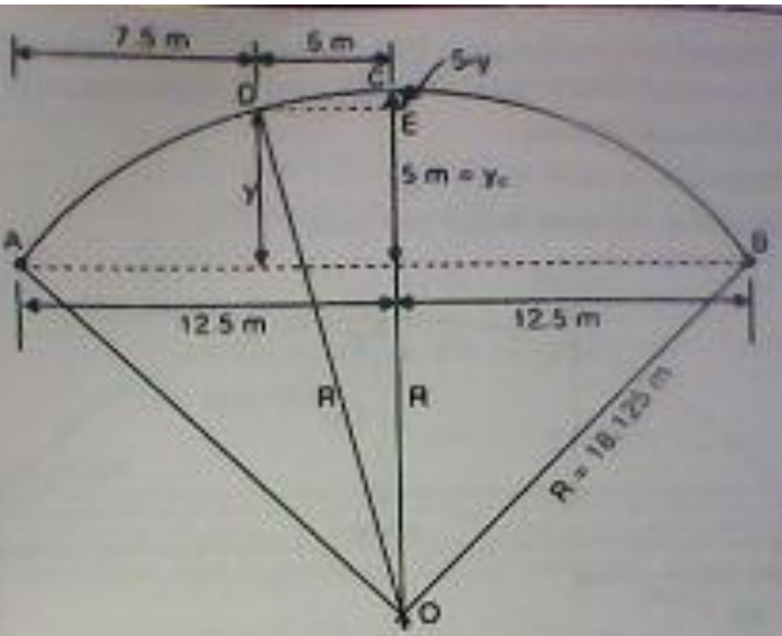


Fig. 4.16

...ove Figure, ΔODE is a right angled triangle

$$OD^2 = OE^2 + ED^2 = (OC - CE)^2 + ED^2$$

$$R^2 = (R - (5 - y))^2 + 5^2$$

$$(18.125)^2 = (18.125 - 5 + y)^2 + 25$$

$$303.515 = (13.125 + y)^2$$

$$\sqrt{303.515} = 13.125 + y$$

$$17.421 = 13.125 + y$$

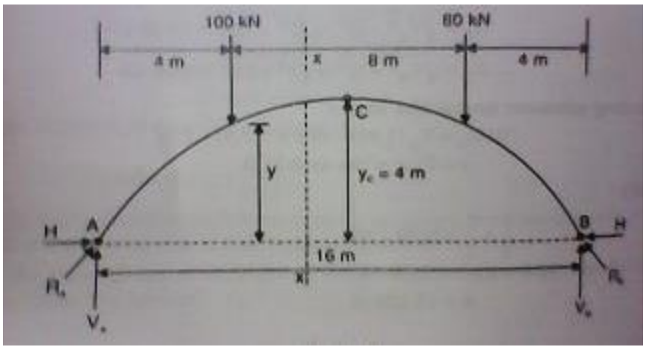
$$y = 4.3 \text{ m} \text{ ; substitute in (i)}$$

$$(BM)_D = 7(7.5) - 7.5\,(4.3)$$

$$(BM)_D = 20.25 \text{ kNm.}$$

A three hinged circular arch of span 16m and rise 4m is subjected to two point loads of 100 kN and 80 kN at the left and right quarter span points respectively. Find the reactions at supports. Find also the bending moment, radial shear and normal thrust at 6m from left support.

Solution:



(a) Reaction at A and B:

(i) Vertical components of reactions at A and B:

Taking moment about A,

$100\,(4) + 80(12) - V_B(16) = 0$

$V_B = 85$ kN.

$V_A =$ Total load $- V_B = (100+80)-85$

$V_A = 95$ kN.

b. Horizontal components of reactions at A and B;

Taking moments about the crone points C

$V_A(8) - H(Y_C) - 100(4) = 0$

$95\,(8) - H\,(y_C) - 100\,(4) = 0$

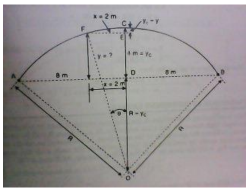
$H = 90$ kN

(iii) Resultant reactions at A and B:

$$R_A = \sqrt{V_A^{\,2} + H^2} = \sqrt{95^2 + 90^2} = 130.86\ kN$$

$$R_B = \sqrt{V_B^{\,2} + H^2} = \sqrt{85^2 + 90^2} = 123.79\ kN$$

(b) Bending moment at 6m from the left support:



In fig. ΔOEF is a right-angled triangle

To find the radius R,

$$\frac{l}{2} * \frac{l}{2} = y_c\ (2R - y_c)$$

$y_c = 4$: L = 16

Therefore R = 10 m

To find y at x = 2m from center:

ΔOEF is a right-angled triangle

$R^2 = (R - 4 + y)^2 + 2^2$

$Y^2 + 12\ y - 60 = 0$

By solving this equ

Y = 3.8m at x = 2m from center

Bending moment = V_A (6) – H_A (y) – 100 (2)

= 95 (6) – 90y -100 (2)

BM = 28 kNm

(c) Radial shear force 'F' :

From fig.

$$\theta = \tan^{-1}\frac{2}{OE} = \tan^{-1}\frac{2}{6+y} = \tan^{-1}\frac{2}{6+3.8} = 11°32'$$

R = V cos θ - H sin θ

V = net shear force at x = 6m from A

= V_A -100 = 95 – 100 = -5 kN

H = 90 kN

R = -5 cos (11º32') – 90 sin (11º32') = - 22.895

R = -22.89 kN

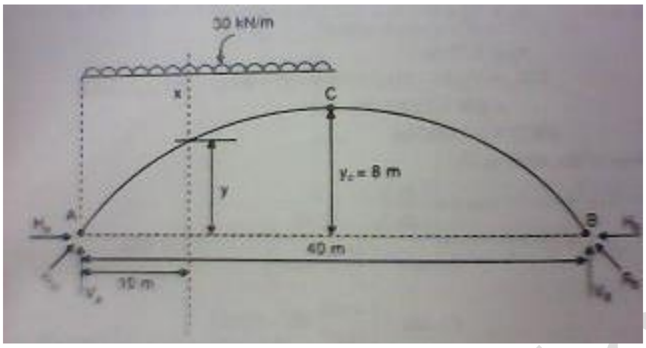
(D) Normal Thrust (at x = 6m from A)

N = V sin θ + H cos θ

N = -5 sin (11º32') + 90 cos (11º32') = -87.17 kN

A symmetrical three hinged parabolic arch of span 40m and rise 8m carries an udl of 30 kN/m over left of the span. The hinges are provided at these supports and at the center of the arch. Calculate the reactions at the supports. Also calculate the bending moment, radial shear, normal thrust at distance of 10 m from the left support.

Solution:



(1) Reactions at the supports:

(i) Vertical components;

Taking moments about A,

$$30*\frac{(20)^2}{2} - V_B*(40) = 0$$

Vertical component of R_B, $V_B = 150$ kN

V_A = Total load – V_B = 30 * 20 – 150 = 450 kN

(iii) Horizontal components

Taking moments about the crown, 'C',

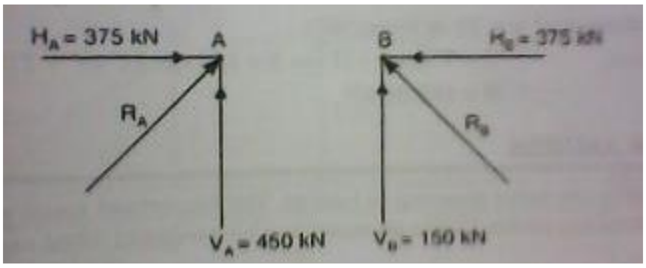
$$V_A\left(\frac{l}{2}\right) - H_A(y_c) - 30*20*\frac{20}{2} = 0$$

$$450(20) - H_A(8) - 30*\frac{(20)^2}{2} = 0$$

$H_A = 375$ kN

$\sum H = 0$; $H_A - H_B = 0$

(iii) Reaction R_A and R_B :



$$R_A = \sqrt{V_A^2 + H_A^2} = \sqrt{(450)^2 + (375)^2} = 585.77\ kN$$

$$R_B = \sqrt{V_B^2 + H_B^2} = \sqrt{(150)^2 + (375)^2} = 403.89\ kN$$

(2) Bending moment at 10 m from A:

$$y = \frac{4r}{l^2} x(l - x)$$

$$y = \frac{4 * 8}{(40)^2} * 10(40 - 10)$$

$$y = 6m \ at \ 10m \ from \ A.$$

Bending moment $= + V_A(10) - H_A(y) - 30 * 10 * \frac{10}{2}$

= 450(10) – 375y – 30 (50)

BM_{xx} = 3000 -375y

BM_{xx} = 3000 – 375 (6)

BM at 10m = 750 kNm

(3)Radial shear force at x = 10m:

R = Radial shear force = V cos θ – H sin θ

Where, V = Net vertical shear force at x = 10m from A

H = Horizontal thrust.

$$\theta = \tan^{-1}\left(\frac{4r}{l^2}(l - 2x)\right)$$

- $\tan\theta = \frac{dy}{dx} = \frac{d}{dx}\left(\frac{4r}{l^2} x(l - x)\right)$

$$\tan\theta = \left(\frac{4r}{l^2}(l - 2x)\right)$$

$$\theta = \tan^{-1}\left(\frac{4 * 8}{(40)^2}(40 - 2(10))\right)$$

θ = 21º 48’

V = V_A – wl/4 = 450 – 30* 10

Radial shear force, R = V cos θ – H sin θ

R = 150 cos 21º 48’ – 375 sin 21º48’

R = 0

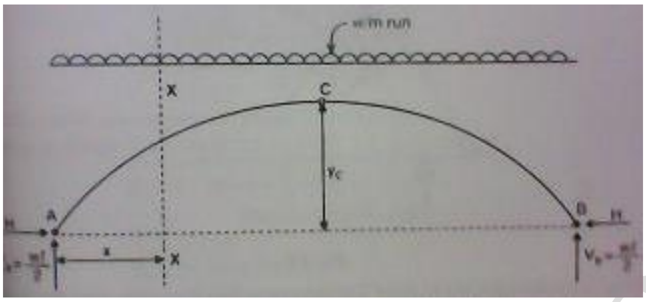
(4) Normal thrust at x = 10m from 'A':

Normal thrust, N = V sin θ + H cos θ = 150 sin 21º48' + 375 cos 21θ 48'

N = 403.89 kN

A parabolic 3-hinged arch of span 'l' is subjected to an u.d.l of w/m run over the entire span. Find the horizontal thrust and bending moment at any section XX.

Solution:



(a) Reactions (Vertical) at the supports:

As the loading is symmetrical, vertical reactions at A and B are equal

$V_A = V_B$ = Total load/2 = wl/2

(b) Horizontal thrust:

Taking moments about the crown point C,

$$V_A\left(\frac{l}{2}\right) - \frac{w\left(\frac{l}{2}\right)^2}{2} - H(y_c) = 0$$

$$\frac{wl}{2} * \frac{l}{2} - \frac{wl^2}{4*2} - H(y_c) = 0$$

$$\frac{wl^2}{4} - \frac{wl^2}{8} - H(y_c) = 0$$

$$H(y_c) = \frac{wl^2}{8}$$

$$H = \frac{wl^2}{8y_c}$$

(c) Bending moment at xx;

$$M_x = V_A(x) - \frac{wx^2}{2} - Hy_x$$

Since the arch is parabolic,

$$y = \frac{4\,y_c}{l^2}\,x(l-x)$$

$$M_x = \frac{wl}{2}x - \frac{w}{2}x^2 - \frac{wl^2}{8y_c}\cdot\frac{wl^2}{8y_c}\cdot\frac{4\,y_c}{l^2}.x(l-x)$$

$$= \frac{wlx}{2} - \frac{wx^2}{2} - \frac{wxl}{2} + \frac{wx^2}{2} = 0$$

CE6501- STRUCTURAL ANALYSIS – CLASSICAL METHOD
(FOR V – SEMESTER)

UNIT - II

UNIT – II

TWO MARKS QUESTIONS AND ANSWERS

Influence lines

An influence line is a graph showing, for any given beam frame or truss, the
variation of any force or displacement quantity (such as shear force, bending moment, tension, deflection) for all positions of a moving unit load as it crosses the structure from one end to the other.

Uses of influence line diagram

(i) Influence lines are very useful in the quick determination of reactions, shear force, bending moment or similar functions at a given section under any given system of moving loads and

(ii) Influence lines are useful in determining the load position to cause maximum value of a given function in a structure on which load positions can vary.

A simply supported beam of span 10m carries a udl of 20 kN/m over its central 4m length. With the help of influence line diagram, find the shear force at 3m from the left support.

$$\frac{x}{l} = \frac{3}{10} = 0.3$$

$$\frac{(l-x)}{l} = \frac{7}{10} = 0.7$$

Shear force at X = intensity of udl x area of udl below the udl

$$= 20 * \frac{(0.7+0.3)}{2} * 4 = 40\ kN$$

Muller – Bresalu principle.

Muller – Bresalu principle states that, if we want to sketch the influence line for any force quantity (like thrust, shear, reaction, support moment or bending moment) in a structure,

(i) we remove from the structure the restraint to that force quantity

(ii) we apply on the remaining structure a unit displacement corresponding to that forces quantity.

For the two member determine bent in fig. sketch the influence line for V_A.

If we deftly apply Muller – Breslau to the problem, we can first remove support A and push A up by unit distance. Since the angle at B will remain unchanged, our unit displacement will result in a rigid body rotation of θ (=1/4 radian) about C. So the column CB will also have a horizontal displacement. (For the part AB, the diagram is just the influence line diagram for shear in a S.S beam)

The diagram for BC must be understood to be the influence of horizontal loads on the column on R_A.

principle on which indirect model analysis is based

The indirect model analysis is based on the Muller Bresalu principle.

Muller Breslau principle has lead to simple method of using models of structures to get the influence lines for force quantities like bending moments, support moments, reactions, internal shears, thrusts, etc.

To get the influence line for any force quantity (i) remove the restraint due to the force, (ii) apply a unit displacement in the direction of the force.

Begg’s deformeter

Begg’s deformeter is a device to carry out indirect model analysis on structures. It has the facility to apply displacement corresponding to moment, shear or thrust at any desired point in the model. In addition, it provides facility to measure accurately the consequent displacements all over the model.

‘dummy length’ in models tested with Begg’s deformeter.

Dummy length is the additional length (of about 10 to 12 mm) left at the extremities of the model to enable any desired connection to be made with the gauges.

Three types of connections possible with the model used with Begg’s deformeter.

(i) Hinged connection
(ii) Fixed connection
(iii) Floating connection

Use of a micrometer microscope in model analysis with Begg’s deformeter

Micrometer microscope is an instrument used to measure the displacements of any point in the x and y directions of a model during tests with Begg’s deformeter.

Name the types of rolling loads for which the absolute maximum bending moment occurs at the mid span of a beam.

Types of rolling loads:

(i) Single concentrated load
(ii) Udl longer than the span
(iii) Udl shorter than the span

Absolute maximum bending moment in a beam

When a given load system moves from one end to the other end of a girder, depending upon the position of the load, there will be a maximum bending moment for every section. The maximum of these maximum bending moments will usually occur near or at the mid span. This maximum of maximum bending moment is called the absolute maximum bending moment, $M_{maxmax.}$

The portal frame in fig. is hinged at D and is on rollers at A. Sketch the influence line for bending moment at B.

To get the influence line diagram for M_B, we shall introduce a hinge at B (and remove the resistance to bending moment). Now we get a unit rotation between BA and BC at B.

BC cannot rotate since column CD will prevent the rotation. BA would rotate freely (with zero moment). For θ =1 at B, displacement at A = 3m. The displaced position shows the influence line for M_B as shown in fig.

A single rolling load of 100 kN moves on a girder of span 20m. (a) Construct the influence lines for (i) shear force and (ii) bending moment for a section 5m from the left support. (b) Construct the influence lines for points at which the maximum shears and maximum bending moment develop. Determine these values.

Solution:

(a) To find maximum shear force and bending moment at 5m from the left support:

For the ILD for shear,

IL ordinate to the right of D = $\frac{l-x}{l} = \frac{20-5}{20} = 0.75$

For the IL for bending moment, IL ordinate at D =

$\frac{x(l-x)}{l} = \frac{5*15}{20} = 3.75\ m$

(i) Maximum positive shear force

By inspection of the ILD for shear force, it is evident that maximum positive shear force occurs when the load is placed just to the right of D.

Maximum positive shear force = load * ordinate = 100 * 7.5

At D, SF_{max} + = 75 kN.

(ii) Maximum negative shear force

Maximum negative shear force occurs when the load is placed just to the left D.

Maximum negative shear force = load * ordinate = 100 * 0.25

At D, SF_{max} = -25 kN.

(iii) Maximum bending moment

Maximum bending moment occurs when the load is placed on the section D itself.

Maximum bending moment = load * ordinate = 100 * 3.75 = 375 kNm

(b) Maximum positive shear force will occur at A. Maximum negative shear force will occur at B. Maximum bending moment will occur at mid span. The ILs are sketched in fig.

(i) Positive shear force

Maximum positive shear force occurs when the load is placed at A.

Maximum positive shear force = load * ordinate = 100*1

SF_{maxmax} + = 100 kN

(ii) Negative shear force

Maximum negative shear force occurs when the load is placed at B.

Maximum negative shear force = load * ordinate = 100 * (-1)

SF_{maxmax} = - 100 kN

(iii) Maximum bending moment

Maximum bending moment occurs when the load is at mid span

Maximum bending moment = load * ordinate = 100 * 5 = 500 kNm

Draw the ILD for shear force and bending moment for a section at 5m from the left hand support of a simply supported beam, 20m long. Hence, calculate the maximum bending moment and shear force at the section, due to a uniformly distributed rolling load of length 8m and intensity 10 kN/m run.(Apr/May 05)

Solution:

(a) Maximum bending moment:

Maximum bending moment at a D due to a udl shorter than the span occurs when the section divides the load in the same ratio as it divides the span.

In the above fig. $\frac{A_1 D}{B_1 D} = \frac{AD}{BD} = 0.25, A_1 D = 2M, B_1 D = 6M$

Ordinates:

Ordinate under A_1 = (3.75/5)*3 = 2.25

Ordinate under B_1 = (3.75/15)*9 = 2.25

Maximum bending moment = Intensity of load * Area of ILd under the load

$$= 10 * \frac{(3.75 + 2.25) * 8}{2}$$

At D, M_{max} = 240 kNm

(b) Maximum positive shear force

Maximum positive shear force occurs when the tail of the UDL is at D as it traverses from left to right.

$$\text{Ordinate under } B_1 = \frac{0.75}{15} * (15 - 8) = 0.35$$

Maximum positive shear force = Intensity of load * Area of ILD under load

$$= 10 * \frac{(0.75 + 0.35) * 8}{2}$$

SF_{max} = + 44 kNm

(c) Maximum negative shear force

Maximum negative shear force occurs when the head of the UDL is at D as it traverses from left to right.

Maximum negative shear force = Intensity of load * Area of ILD under the load

= 10(1/2*0.25*5)

Negative SF_{max} = 6.25 kN.

Two point loads of 100 kN and 200 kN spaced 3m apart cross a girder of span 15m from left to right with the 100 kN load loading. Draw the influence line for shear force and bending moment and find the value of maximum shear force and bending moment at a section, 6m from the left hand support. Also, find the absolute maximum moment due to the given load system.

Solution:

(a) Maximum bending moment

Max. Ordinate of ILD = $\dfrac{x(l-x)}{l} = \dfrac{6 * 9}{15} = 3.6\ m$

Maximum bending moment at D occurs under the critical load. This load, when it moves from left to right of 'D' changes the sign of L_r, the differential loading rate, where,

$$L_r = \frac{W_{left}}{x} - \frac{W_{right}}{l-x}$$

Now, let us try with 200 kN load. Firstly, keep this 200 kN load to the left of 'D'.

$$L_r = \frac{W_{left}}{x} - \frac{W_{right}}{l-x} = \frac{200}{6} - \frac{100}{9} = 22.22\ (+ve)$$

Moving this load to the right of 'D',

$$L_r = \frac{W_{left}}{x} - \frac{W_{right}}{l-x} = \frac{0}{6} - \frac{300}{9} = 33.33\ (-ve)$$

Since, the sign of L_r changes, the 200 kN load is critical. The maximum bending moment occurs at D when this 200 kN load is placed at D.

Ordinate under 100 kN load = $\frac{3.6}{9} * 6 = 2.4$

Maximum bending moment = $\sum(load * ordinate) = 200 * 3.6 + 100 * 2.4$

(b) Maximum shear force:

(i) Positive shear force

Fig. shows the load position for the absolute maximum positive shera force. If the load train is moved to the left, the positive contribution due to the bigger (160 kN) load is lost and a negative contribution is obtained.

If the load train moves to the right, the ordinates under to the loads decrease. Hence, the indicated load position is the critical one.

Ordinate under 200 kN load = 0.6

Ordinate under 100 kN load = $\frac{0.6}{9} = 0.4$

Maximum positive shear force = 200 (0.6) + 100 (0.4)

SF_{max} = + 160 kN.

(ii) Negative shear force

Trying with 100 kN load, first keep this 100 kN load to the left of D. Then move this load to the right of ‘D’ by 3m. If the value of the shear increment S_i is a negative value, it includes a decrease in negative shear force.

$$S_i = \frac{W_c}{l} - W_1 - \frac{300 * 3}{15} - 100 = -40(-ve)$$

Hence, the negative shear force decreases when this load train is moved to the right of ‘D’. Hence, to get maximum shear force, this 100 kN load should be kept just to the left of D.

Ordinate under 100 kN load = 0.4

Ordinate under 200 kN load = $\frac{0.4}{6} * 3 = 0.2$

Maximum negative shear force = 100*0.4 + 200*0.2 = 80

SF_{max} = - 80 kN

(c) Absolute maximum bending moment

(i) Resultant of the loads

Taking moments about 200 kN load,

100*3 = R.x, R = 300 kN. X = 1.0 m

Absolute maximum bending moment occurs under the load which is nearer to the resultant ‘R’. The critical position is when the resultant ‘R’ and the load are at equal distance from the centre of span © .

Distance of this 200 kN from C = Distance of ‘R’ from c.

Maximum ordinate of ILD (i.e) ordinate under 200 kN load = $\frac{x(l-x)}{l} = 3.733\ m$

Ordinate under 100 kN load = (3.733*6)/9 =2.489 m

Absolute maximum bending moment, $M_{maxmax} = \sum(load * Dis\tan ce)$

= 200 * 3.733 + 100*2.489 = 995.5 kNm

A train of 5 wheel loads crosses a simply supported beam of span 22.5 m. Using influence lines, calculate the maximum positive and negative shear forces at mid span and absolute maximum bending moment anywhere in the span.(Nov/Dec 05)

Solution:

(a) Maximum shear force

(i) Positive shear force

To determine the load position to get the maximum positive shear force, let us keep all the loads to the right of C. Then move W_1 load to the left of 'C' by 2.5 m. if the sign of shear increments S_i is negative, it will indicate that W_1 shall be retained at C.

$$S_i = \frac{W_c}{l} - W_1$$

W = Total load on the span = 120 + 160 +400 + 260 +240 = 1182 kN.

C = Distance through which the load train is moved = 2.5 m

$$S_i = \frac{1180 * 2.5}{22.5} - 120 = 11.11\ (+ve)$$

Since S_i is positive, the shear force increases due to thr shifting of W_1 to the left of C. Again, let us move W_2 to the left of C by 2.5 m to check whether the shera force further increases or not.

$$S_i = \frac{W_c}{l} - W_2 = \frac{1180 * 2.5}{22.5} - 160 = -28.89\ \ (-ve)$$

Since S_i is negative, it undicates that to get maximum positive shear force, W_2 should stay just right of C.

Ordinate of ILD

Ordinate under W_1 = $\frac{-0.5}{11.25} * (11.25 - 2.5) = -0.39$

Ordinate under W_2 = $\frac{22.5 - 11.25}{22.5} = 0.5$

Ordinate under W_3 = $\frac{0.5}{11.25} * 8.75 = 0.39$

Ordinate under W_4 = $\frac{0.5}{11.25} * 3.25 = 0.28$

Ordinate under W_5 = $\frac{0.5}{11.25} * 3.75 = 0.17$

Maximum positive shear force = $\sum(laodxordinate)$

= 120 (-0.39) + 160 (0.5) +400 (0.39) +260(0.28) + 240 (0.17)

At C , SF_{max} + = 302.8 kN.

ii) Negative shear force

To determine the position of loads to get the maximum negative shear force, move the loads one by one to the right of C and computer the value of S_i.If S_i becomes negative it will indicate a decrease in negative shear force due to that movement.

First let us move the leading W_5 to the right of C by 2.5 m and calculate S_i

$$S_i = \frac{Wd}{l} - W_5$$

W_5 = 240 kN; d = 2.5 m

$$S_i = \frac{1180x2.5}{22.5} - 240 = -108.89(-ve)$$

Since S_i is –ve it indicates that W_5 should stay just to the left of C.

Ordinates of ILD:

Ordinate under $W_5 = \frac{x}{l} = \frac{11.25}{22.5} = 0.5$

Ordinate under $W_4 = \frac{0.5}{11.25} x(11.25 - 2.5) = 0.39$

Ordinate under $W_3 = \frac{0.5}{11.25} x(11.25 - 5.0) = 0.278$

Ordinate under $W_2 = \frac{0.5}{11.25} x(11.25 - 7.5) = 0.167$

Ordinate under $W_1 = \frac{0.5}{11.25} x(11.25 - 10) = 0.056$

Maximum negative shear force at C = 240(-0.5)+260(-0.39)+(400(-0.278) +160(-0.167)

+120(-0.056)

F_{max} = 366.04 kN.

b) Absolute maximum bending moment

i) Position of resultant of all loads

Taking moments about W_1, 120(0)+160(2.5)+400(5.0)+260(7.5)+240(10.0) =R. $\bar{x}$

R = 1180kN

$\bar{x}$ = 5,72m from W_1

ii) Location of absolute maximum bending moment

Absolute maximum bending moment occurs under the load, which is nearest to the resultant 'R'.(In this problem W_3 is nearest to the resultant 'R').The distance between C and R and the distance between C and W_3 shall be equal.

Distance between R and center of span(C) = ½ (0.72) =0.36 m

In this fig. Shows the IL for bending moment at the critical spot D, 10.89 m fro A.

Ordinates of ILD:

Maximum ordinate of ILD

(i.e) Ordinate under $W_3 = \frac{x(l-x)}{l} = \frac{10.89(22.5-10.39)}{22.5} = 5.62$

Ordinate under $W_2 = \frac{5.62}{10.89} x 8.39 = 4.33$

Ordinate under $W_1 = \frac{5.62}{10.89} x 5.89 = 3.04$

Ordinate under $W_4 = \frac{5.62}{11.61} x 9.11 = 4.41$

Ordinate under $W_5 = \frac{5.62}{11.61} x 6.61 = 3.20$

Absolute maximum bending moment =120 (3.04)+160(4.33)+400(5.62)+260(4.41)+ 240 (3.2)

$M_{max\ max} = 5220.2$ kN.m

5) A girder a span of 18mis simply supported at the ends. It is traversed by a train of loads as shown in fig. The 50 kN load loading. Find the maximum bending moment which can occur (i) under the 200 kN load (ii) Under 50 kN load, using influence line diagrams.

Solution :

a) Maximum bending moment

i) Under 200 kN loads.

To get the maximum bending moment under W_3 the resultant R and W_3 should be at equal distances from the center of the span C.For that the point of action of resultant R should be determined first.

a) Resultant of loads:

R = 450 kN

Taking moments about W_4

$$200(3) + 100(3+2) + 50(3+2+3) = 450\,\bar{x}$$

$$\bar{x} = 3.33 \text{ from } W_4$$

b) **Bending moment under 200 kN load**

Distance between C and 200 kN load = Distance between C and R = 0.33/2 = 0.165 m.

Ordiantes of ILD :

ILD under $W_3 = \dfrac{x(l-x)}{l}$

X= 8.335m

l-x = 18-8.835 = 9.165 m

ILD under $W_3 = \dfrac{8.835 * 9.1365}{18} = 4.50\ m$

ILD under $W_4 = \dfrac{4.5 * 5.835}{8.835} = 2.97\ m$

ILD under $W_2 = \dfrac{4.5 *(3 + 4.165)}{9.165} = 3.52\ m$

$= 2.05\ m$

Bending moment under the 200 kN load

$$= 200\ (4.5) + 100\ (2.97) + 100\ (3.52) + 50\ (2.05) = 1651.5\ \text{kNm.}$$

(ii) Bending moment under the load 50 kN load

To get the maximum bending moment under W_1 , W_2 and R must be at equal distances from the centre of span ©. Distance between C and R = Distance between C and W_1 = ½ (2 – 0.33 + 3) = ½ (4.67) = 2.335 m

Ordinates of ILD:

$$\text{ILD under } W_1 = \frac{x(l-x)}{l} = \frac{11.335 * 6.665}{18} = 4.20\, m$$

$$\text{ILD under } W_2 = \frac{x(l-x)}{l} = \frac{4.2 * 8.335}{11.335} = 3.09\, m$$

$$\text{ILD under } W_3 = \frac{x(l-x)}{l} = \frac{4.2 * 6.335}{11.335} = 2.35\, m$$

$$\text{ILD under } W_4 = \frac{x(l-x)}{l} = \frac{4.2 * 3.335}{11.335} = 1.24\, m$$

Bending moment under the load 50 kN load

= 4.2 (50) + 3.09 (100) +2.35 (200) +1.24 (100) = 1113 kN m.

6. Draw the I.L for reaction at B and for the support moment M_A at A for the propped cantilever in fig. Compute the I.L ordinates at 1.5 m intervals.

Solution:

Remove the restraint due to R_B (remove support B)

Apply a unit displacement (upward).

When $R_B = 1$, then Y_{XB} is the displacement at section x due to unit load applied at B.

$$M_x = -EI\frac{d^2y}{dx^2} = R_B.x = 1.x;\quad EI\frac{d^2y}{dx^2} = -x$$

$$EI\frac{dy}{dx} = -\frac{x^2}{2} + C_1$$

$$EIy = \frac{-x^3}{6} + C_1x + C_1$$

At x= 12, y = 0, dy/dx = 0

Hence , $C_1 = 72$, $C_2 = -576$

$$Y_{XB} = \frac{1}{EI}\left[\frac{-x^2}{6} + 72x - 576\right]$$

$$Y_{BB}\text{ (at x = 0) } = \frac{-576}{EI}$$

When we plot this against x, we get the I.L for R_B.

X_m	0	1.5	3	4.5	6	7.5	9	10.5	12
R_B	1	0.814	0.632	0.463	0.312	0.184	0.085	0.022	0.00

Fig. is the influence line diagram for R_B.

To get the I.L for M_A have to

(i) Introduce a hinge at A and

(ii) We have to apply a unit rotation at A. Instead we will apply a unit moment at A find the general displacement at any x from B. We will then divide the displacement by the actual rotation at A.

Due to

$$M_A = 1$$

$$R_B = -R_A = 1/12$$

$$M_x = -EI\frac{d^2y}{dx^2} = \frac{x}{12}$$

$$Ei\frac{dy}{dx} = \frac{-x^2}{24} + C_1$$

The ordinates of the I.L.D for M_A at 1.5 m intervals are tabulated below.

X	0	1.5	3.0	4.5	6.0	7.5	9.0	10.5	12.0
ILD	0.0	-0.738	-1.406	-1.934	-2.250	-2.285	-1.960	-1.230	0.0

7. In a simply supported girder AB of span 2om, determine the maximum bending moment and maximum shear force at a section 5m from A, due to the passage of a uniformly distributed load of intensity 20 kN/m, longer than the span.

Solution:

(i) Maximum bending moment

Since the udl is longer than the , the criterion for maximum bending moment at a section is that the whole span should be loaded as shown in fig.

(ii) Maximum shear force

Maximum negative shear force at a section occurs when the head of the load reaches the section ((i.e. when the left portion AX is loaded and right portion XB is empty)

(iii) Maximum positive shear force:

Maximum positive shear force occurs at X when the tail of the load is at X as it moves from left to right. (i.e. AX is empty and the portion XB is loaded)

Maximum positive shear force = R_A