Ahmed Badr

lec 2

Ex 1

$$y' = \frac{1+y^2}{1+x^2}$$

$$\frac{dy}{dx} = \frac{1}{1+x^2}(1+y^2)$$

$$\frac{1}{1+y^2}\,dy = \frac{1}{1+x^2}\,dx$$

$$\int \frac{1}{1+y^2}\,dy = \frac{1}{1+x^2}\,dx$$

$$\tan^{-1} y = \tan^{-1} x + c$$

$$y = \tan(\tan^{-1} x + c)$$

Ex 2

$$x(1-y^2)\,y' + (1+x^2)\,y = 0 \quad \& \quad y(\tfrac{1}{2}) = 2$$

$$x(1-y^2)\,y' = -(1+x^2)\,y \Longrightarrow y' = \frac{-(1+x^2)\,y}{x(1-y^2)}$$

$$\frac{dy}{dx} = \frac{-(1+x^2)}{x} * \frac{y}{(1-y^2)}$$

$$\int\left(\frac{1-y^2}{y}\right)dy = \int\left(-\frac{1+x^2}{x}\right)dx$$

$\Rightarrow \int \left(\frac{1}{y} - y\right) dy = \int \left(-\frac{1}{x} - x\right) dx$

$\Rightarrow \ln y - \frac{y^2}{2} = -\ln x - \frac{x^2}{2} + c \rightarrow$ General solution

$x = \frac{1}{2}$, $y = 2$

$\ln 2 - 2 = -\ln\left(\frac{1}{2}\right) - \frac{1}{8} + c \Rightarrow c = \frac{-15}{8}$

$\therefore \ln y - \frac{y^2}{2} = -\ln x - \frac{x^2}{2} - \frac{15}{8} \rightarrow$ Particular solution

* note *

if $\frac{dy}{dx} = f(ax' + by')$

we put $z = ax + by \xRightarrow{\text{diff}} \frac{dz}{dx} = a + b\frac{dy}{dx}$

$\therefore \frac{1}{b}\left[\frac{dz}{dx} - a\right] = f(z)$

Ex.

$y' = \cos(x+y)$

$z = x + y \xRightarrow{\text{diff}} \frac{dz}{dx} = 1 + \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{dz}{dx} - 1$

$\Rightarrow \cos z = \frac{dz}{dx} - 1 \Rightarrow \frac{dz}{dx} = \cos z + 1$

$\therefore \int \frac{1}{1+\cos z} dz = \int dx \xRightarrow{* \frac{1-\cos z}{1-\cos z}} \int \frac{1-\cos z}{\sin^2 z} dz = x + c$

$= \int \sec^2 z - \cot z \left(\frac{1}{\sin z}\right) dz = x + c$

أو حل آخر بالمتطابقات

Ex.

$$y' = \sqrt{3x - 2y + 1}$$

$$Z = 3x - 2y + 1 \longrightarrow \frac{dz}{dx} = 3 - 2\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[3 - \frac{dz}{dx}\right] \Rightarrow \sqrt{Z} = \frac{1}{2}\left[3 - \frac{dz}{dx}\right]$$

$$3 - \frac{dz}{dx} = 2\sqrt{z} \Rightarrow \frac{dz}{dx} = 3 - 2\sqrt{z}$$

لازالة الجذر $\sqrt[m]{F}$
نعوض $U^m = F$

$$\int \frac{1}{3 - 2\sqrt{z}}\, dz = \int dx$$

$u^2 = z$

$2u\,du = dz$

$$\Rightarrow \int \frac{2u}{3 - 2u}\, du = x + c$$

$$\Rightarrow -\int \boxed{\frac{-2u + 3}{3 - 2u}} - \frac{3}{3-2u}\, du = x + c$$

$$\Rightarrow -\int \left(1 - \frac{3}{3 - 2u}\right) du = x + c$$

$$-u - \frac{3}{2}\ln(3 - 2u) = x + c$$

$$-\sqrt{z} - \frac{3}{2}\ln(3 - 2\sqrt{z}) = x + c$$

$$-\sqrt{3x - 2y + 1} - \frac{3}{2}\ln\left(3 - 2\sqrt{3x - 2y + 1}\right) = x + c$$

$$\sqrt{3x - 2y + 1} + \frac{3}{2}\ln\left(3 - 2\sqrt{3x - 2y + 1}\right) + x = -c$$

2nd Type (Homogeneous Equations)

$$\frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

خطوات الحل

* $z = \frac{y}{x} \Longrightarrow \frac{dy}{dx} = f(z)$
* $y = z\,x \xrightarrow{\frac{d}{dx}} \frac{dy}{dx} = z + x\frac{dz}{dx}$
* $f(z) = z + x\frac{dz}{dx} \Longrightarrow \frac{dz}{dx} = \left(\frac{1}{x}\right)\left[f(z) - z\right]$

Ex.

$$x(x+y)\,y' = x^2 + y^2$$

$$y' = \frac{dy}{dx} = \frac{x^2+y^2}{x^2+xy} \Longrightarrow \frac{dy}{dx} = \frac{1+\frac{y^2}{x^2}}{1+\frac{y}{x}}$$

$$z = \frac{y}{x} \Longrightarrow y = xz \Longrightarrow \frac{dy}{dx} = z + x\frac{dz}{dx}$$

$$z + x\frac{dz}{dx} = \frac{1+z^2}{1+z} \Longrightarrow x\frac{dz}{dx} = \frac{1+z^2}{1+z} - z$$

$$\frac{dz}{dx} = \frac{1}{x}\left[\frac{1+z^2}{1+z} - z\right] = \frac{1}{x}\left[\frac{1-z}{1+z}\right]$$

$$\int \frac{1+z}{1-z}\,dz = \int \frac{1}{x}\,dx \Longrightarrow -\int \frac{1-z-2}{1-z}\,dz = \ln x + \ln c$$

$$\rightarrow -\int 1 - \frac{2}{1-z}\,dz = -z - 2\ln(1-z) = \ln x + \ln c$$

$$z = \ln\frac{1}{cx(1-v)^2} \Longrightarrow y = x\ln\left(\frac{1}{cx\left(1-\frac{y}{x}\right)^2}\right)$$

Ex. Page 12

$$x y' = \sqrt{x^2 - y^2} + y$$

$$\frac{dy}{dx} = y' = \sqrt{1 - \frac{y^2}{x^2}} + \frac{y}{x}$$

$$Z = \frac{y}{x} \Rightarrow y = xZ$$

$$\frac{dy}{dx} = Z + x\frac{dZ}{dx} \Rightarrow \sqrt{1 - Z^2} + Z = Z + x\frac{dZ}{dx}$$

$$\frac{dZ}{dx} = \left(\frac{1}{x}\right)\left(\sqrt{1 - Z^2}\right)$$

$$\int \frac{1}{\sqrt{1 - Z^2}}\, dZ = \int \frac{1}{x}\, dx$$

$$\sin^{-1} Z = \ln x + \ln c$$

$$\frac{y}{x} \leftarrow Z = \sin(\ln xc)$$

$$\therefore\ y = x \sin(\ln xc)$$

3rd Type

$$\frac{dy}{dx} = \frac{a_1 x + b_1 y + c_1}{a_2 x + b_2 y + c_2}$$

متوازيات

متقاطعات

$\frac{a_1}{b_1} = \frac{a_2}{b_2}$ ① متوازيات

$Z = a_1 x + b_1 y$ or $Z = a_1 x + b_1 y + c$

$Z = a_2 x + b_2 y$ or $Z = a_2 x + b_2 y + c$

Ex 7

$$y' = \frac{x-y-1}{y-x-1} \quad \& \quad y(2)=1$$

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = -1 \longrightarrow$$ خطان متوازيان

$$z = x-y \xrightarrow{\frac{d}{dx}} \frac{dz}{dx} = 1 - \frac{dy}{dx}$$

$$\frac{dy}{dx} = 1 - \frac{dz}{dx} \Longrightarrow \frac{z-1}{-z-1} = 1 - \frac{dz}{dx}$$

$$\frac{dz}{dx} = \frac{2z}{z+1}$$

$$\int \frac{z+1}{z}\,dz = \int 2\,dx$$

$$\int \left(1+\frac{1}{z}\right)dz = 2x + c$$

$$z + \ln z = 2x + c$$

$$x - y + \ln(x-y) = 2x + c$$

$$\ln(x-y) - x - y = c$$

$$y(2) = 1 \Longrightarrow \ln(1) - 1 - 1 = c \Longrightarrow c = -2$$

$$\ln(x-y) - x - y = -2$$

Ahmed Badr